

Seminário de Coisas Legais

A SIMPATIA DE N OSCILADORES, DOIS PÊNDELLOS E UMA PONTE?

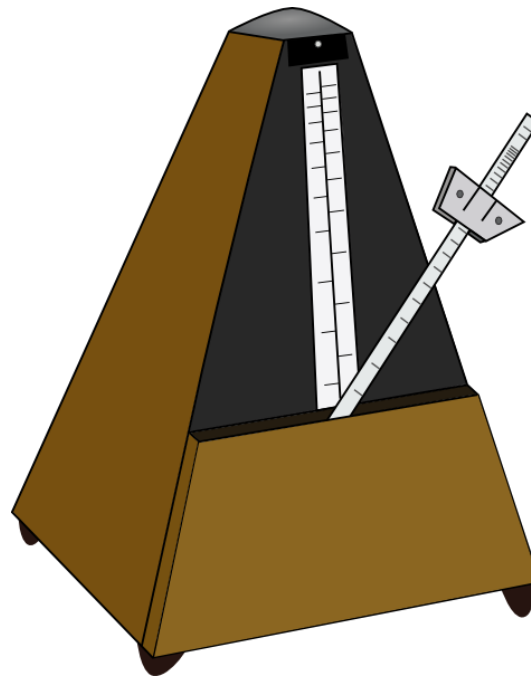
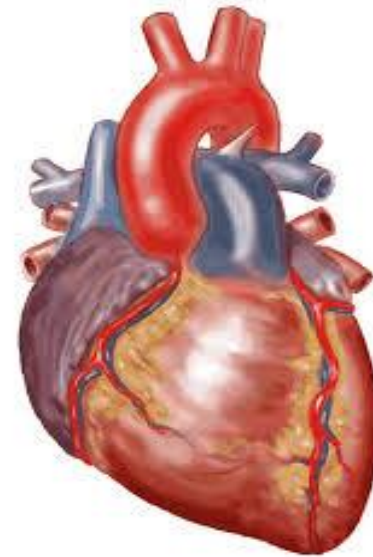
Edmilson Roque dos Santos

Instituto de Física de São Carlos - IFSC

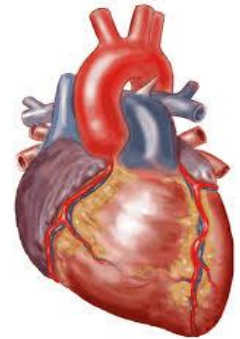
edmilson.roque.usp@gmail.com

O QUE É SINCRONIZAÇÃO?

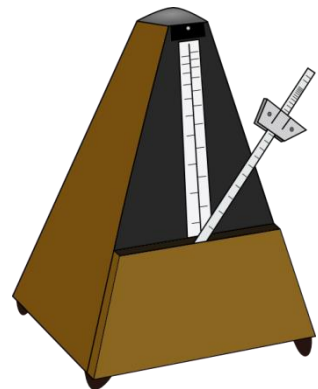
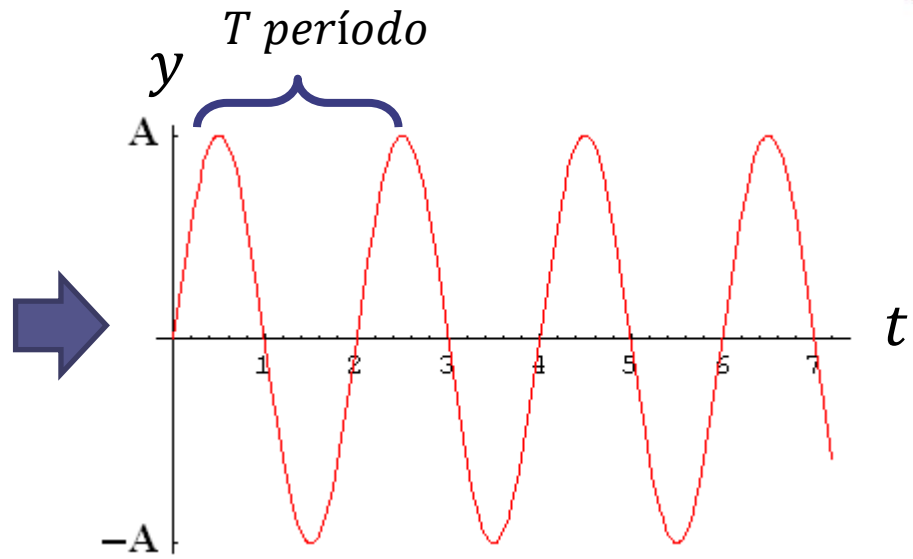
Relaciona-se com objetos oscilantes



O QUE É SINCRONIZAÇÃO?

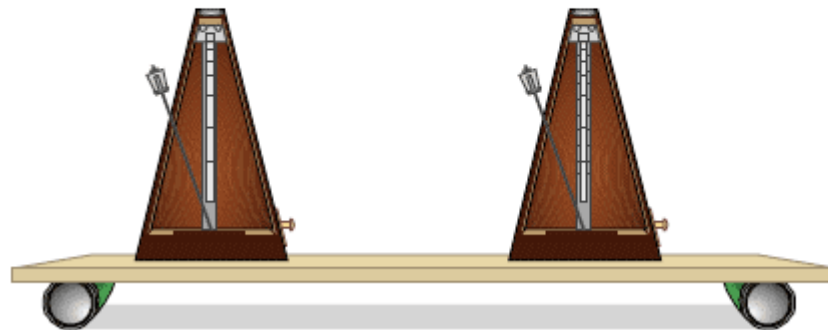
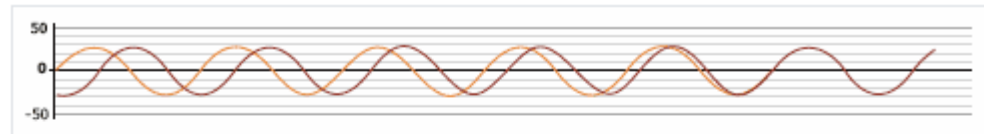
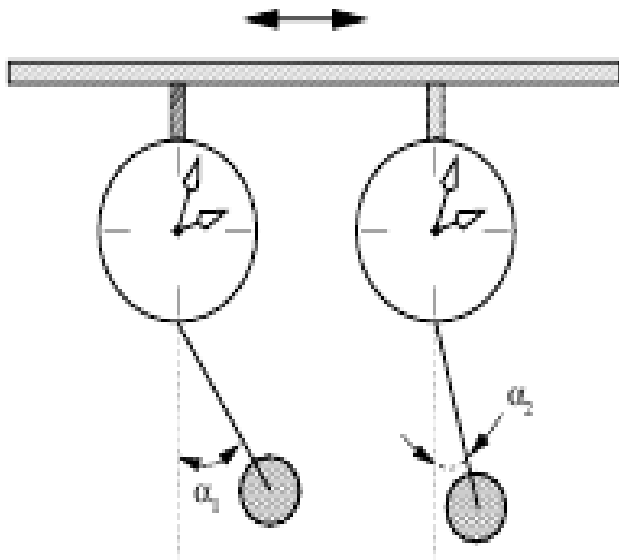


Relaciona –se com objetos oscilantes



O QUE É SINCRONIZAÇÃO?

Relaciona-se com fraca interação

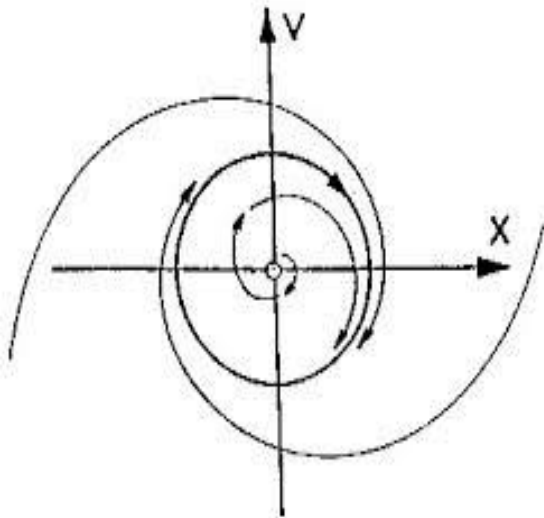


O QUE É SINCRONIZAÇÃO?

“Entendemos sincronização como um **ajustamento** de ritmos de **objetos oscilantes** devido a uma **fraca interação**.” – [M. G. Rosenblum, A. Pikovsky, and J. Kurths. *Synchronization – A universal concept in nonlinear sciences*. 2001.]

“Processos de sincronização são encontrados na natureza e desempenham um papel importante em diferentes contextos como biologia, ecologia, climatologia, sociologia, tecnologia, ou mesmo em artes.” - [Alex Arenas et al. *Synchronization in complex networks*. Physics Reports, 2008.]

Geometricamente, **ciclo-limite**:



Definição:

Osciladores auto-sustentados



EXEMPLOS DE SINCRONIZAÇÃO

Vagalumes



Metrônome

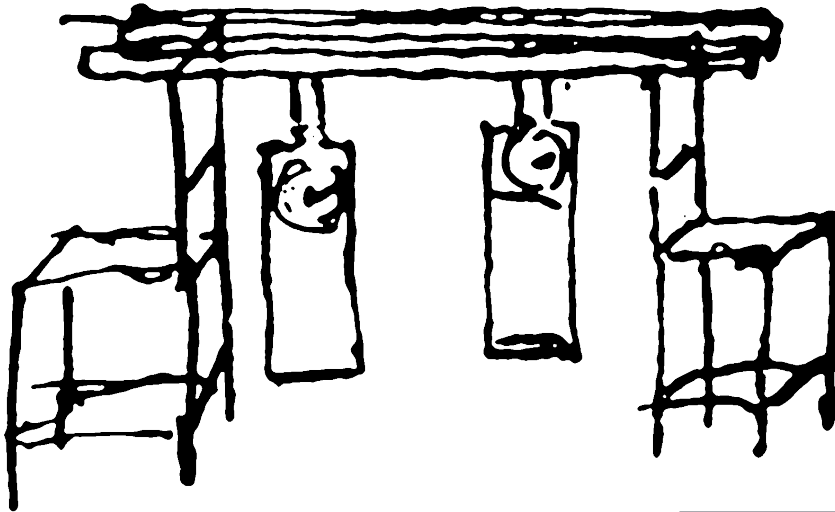


Cardumes e pássaros



DESCOBERTA DO FENÔMENO

“simpatia de dois relógios” (1665)



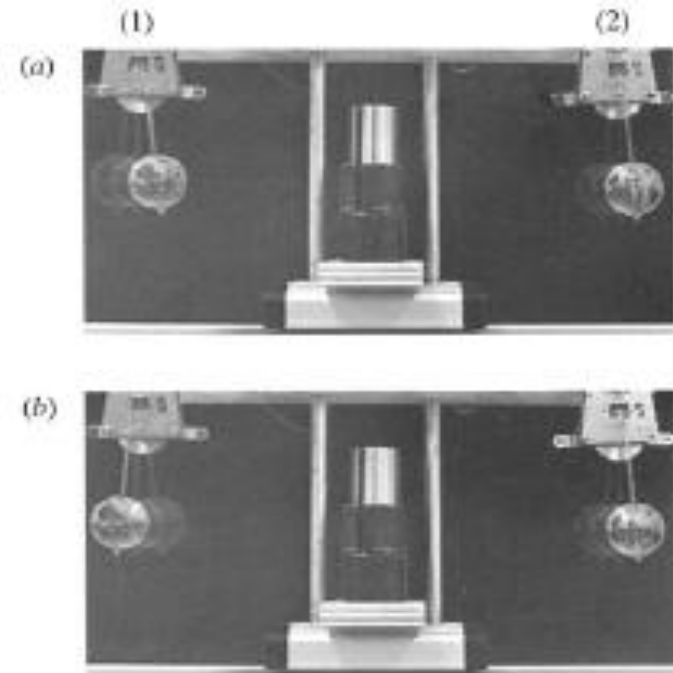
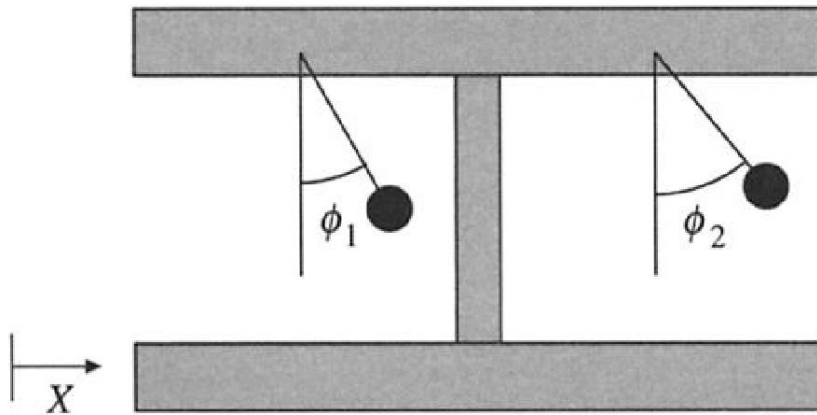
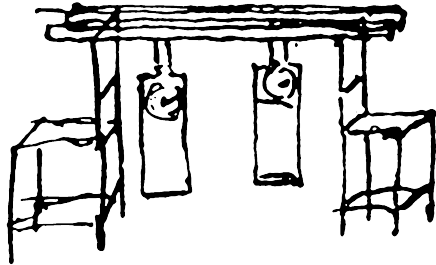
350 anos



Uma resposta para a questão sobre como determinar a longitude no século XVII.

*“The development of an accurate clock would solve this problem, because mariners could then keep track of the time at their home port, and the difference between that time and the local time would tell them their longitude. Huygens's clocks, which tended to lose only **15 seconds a day** (...).”*

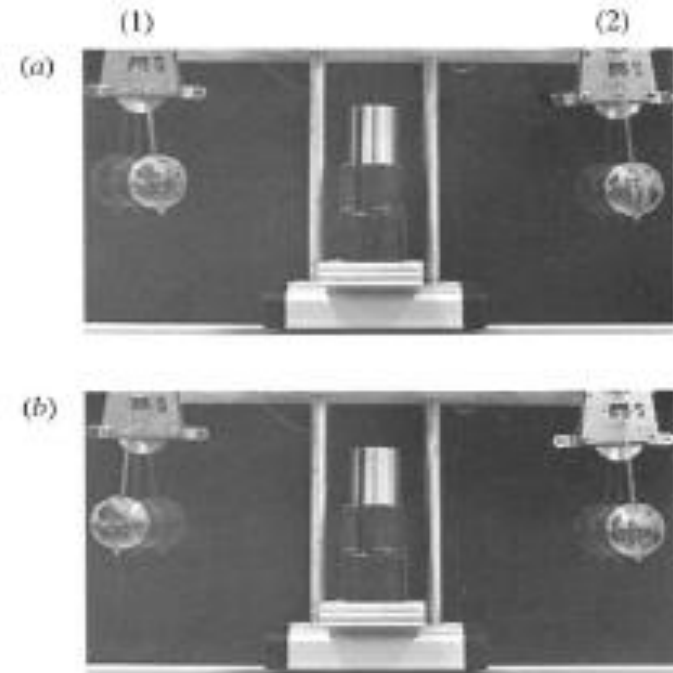
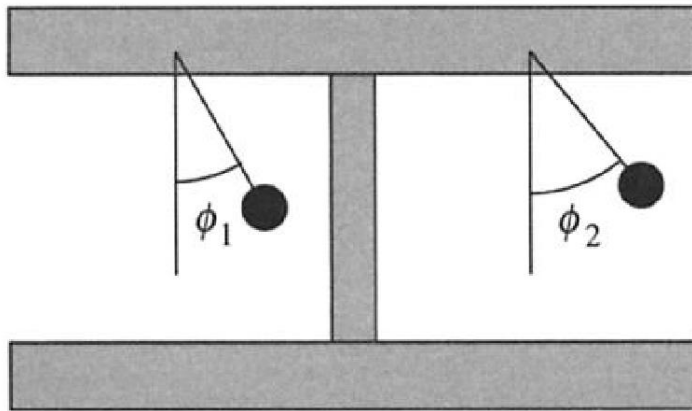
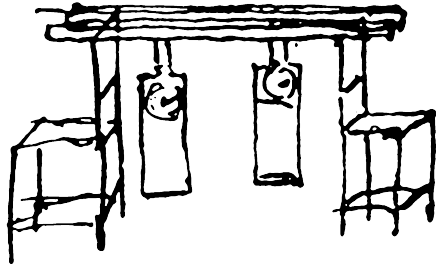
DESCRIÇÃO DO PROBLEMA



$$L = \text{Energia Cinética} - \text{Energia Potencial}$$



DESCRIÇÃO DO PROBLEMA



$$L = T - V$$

$$L = \frac{1}{2}(M + 2m)\dot{X}^2 + m\dot{X}l(\cos\phi_1\dot{\phi}_1 + \cos\phi_2\dot{\phi}_2) + \frac{1}{2}ml^2(\dot{\phi}_1^2 + \dot{\phi}_2^2) + mgl(\cos\phi_1 + \cos\phi_2) - \frac{1}{2}KX^2$$

DESCRIÇÃO DO PROBLEMA

$$L = T - V$$

$$L = \frac{1}{2}(M + 2m)\dot{X}^2 + m\dot{X}l(\cos\phi_1\dot{\phi}_1 + \cos\phi_2\dot{\phi}_2) + \frac{1}{2}ml^2(\dot{\phi}_1^2 + \dot{\phi}_2^2) \\ + mgl(\cos\phi_1 + \cos\phi_2) - \frac{1}{2}KX^2$$

$$Y = X/l$$

$$\tau = t\sqrt{g/l}$$

$$\gamma = b\sqrt{l/4g}$$

$$\Omega^2 = \frac{K}{(M + 2m)}$$

$$\Gamma = \frac{B\sqrt{l/4g}}{(M + 2m)}$$

$$\ddot{\phi}_k + 2\gamma\dot{\phi}_k + \sin\phi_k = -\ddot{Y}\cos\phi_k + f_k$$

$$\ddot{Y} + 2\Gamma\dot{Y} + \Omega^2 Y = -\mu(\sin\phi_1 + \sin\phi_2)''$$

$$\mu = \frac{m}{(M + 2m)}$$

Força de acoplamento*

DESCRIÇÃO DO PROBLEMA

$$\ddot{\phi}_k + 2\gamma\dot{\phi}_k + \sin\phi_k = -\ddot{Y}\cos\phi_k + f_k$$

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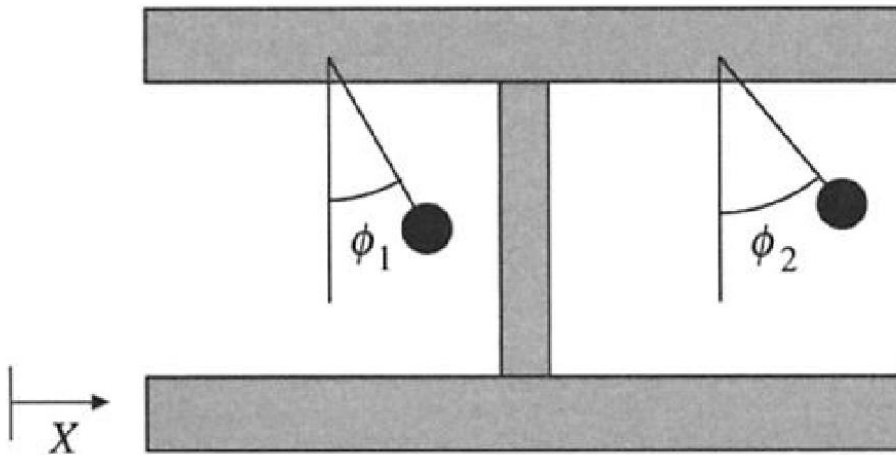
$$\gamma = b\sqrt{l/4g}$$

$$\Omega^2 = \frac{K}{(M + 2m)}$$

$$\Gamma = \frac{B\sqrt{l/4g}}{(M + 2m)}$$

$$\mu = \frac{m}{(M + 2m)}$$

Força de acoplamento*



DESCRIÇÃO DO PROBLEMA

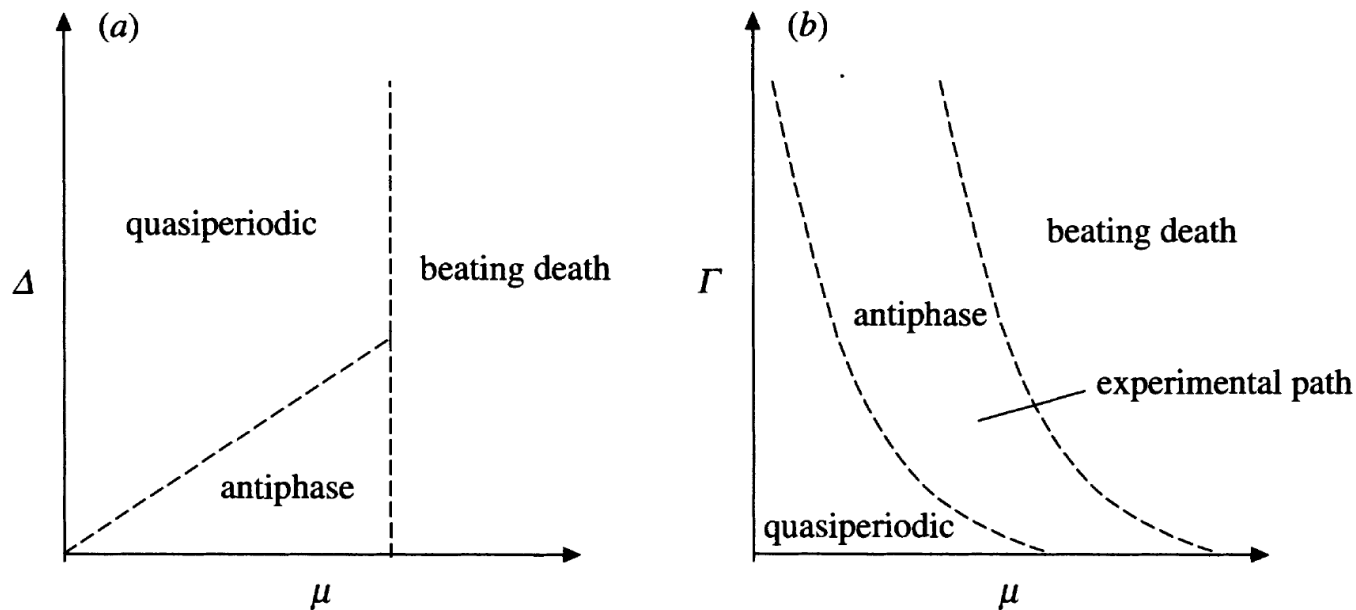


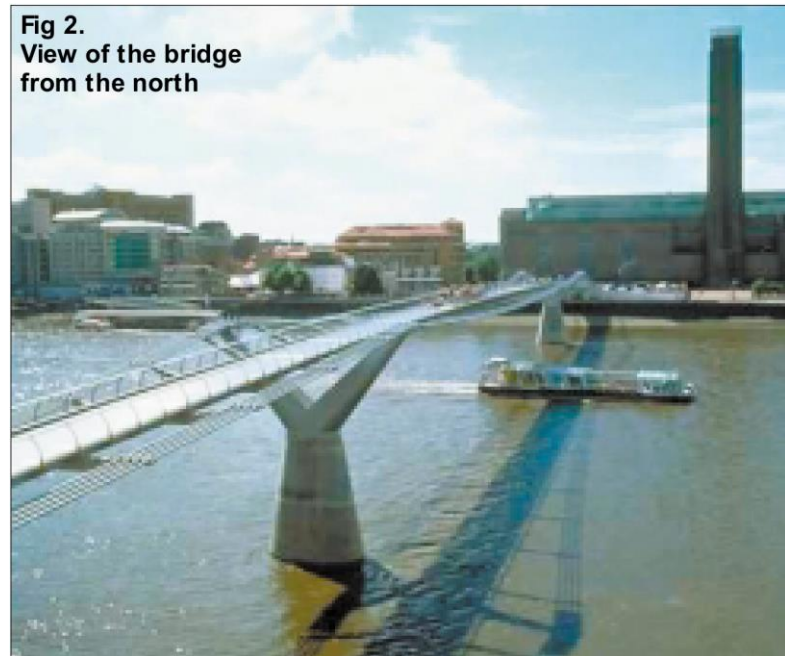
Figure 6. Phase diagrams summarizing theoretical analysis:
(a) Δ - μ parameter plane; (b) Γ - μ parameter plane.

“Our results suggest that Huygens's observation of 'sympathy' depended on both talent and luck.”

MILLENNIUM BRIDGE



Fig 2.
View of the bridge
from the north



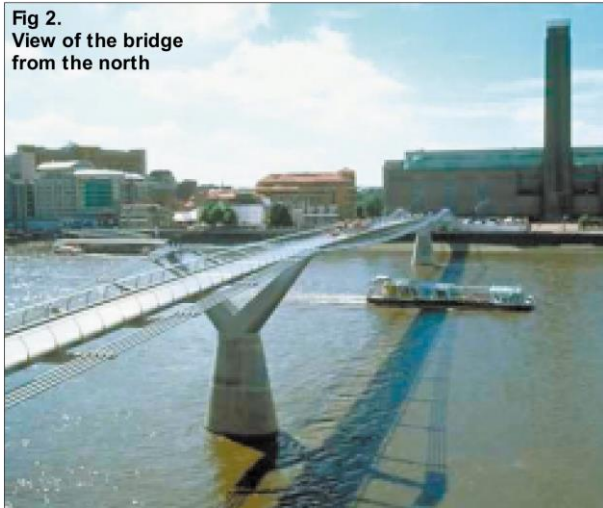
www.londonmillenniumbridge.com*

[Dallard, P. et al. *The London Millennium Footbridge* – Struct. Eng. 6 - 2001]



MILLENNIUM BRIDGE

Fig 2.
View of the bridge
from the north

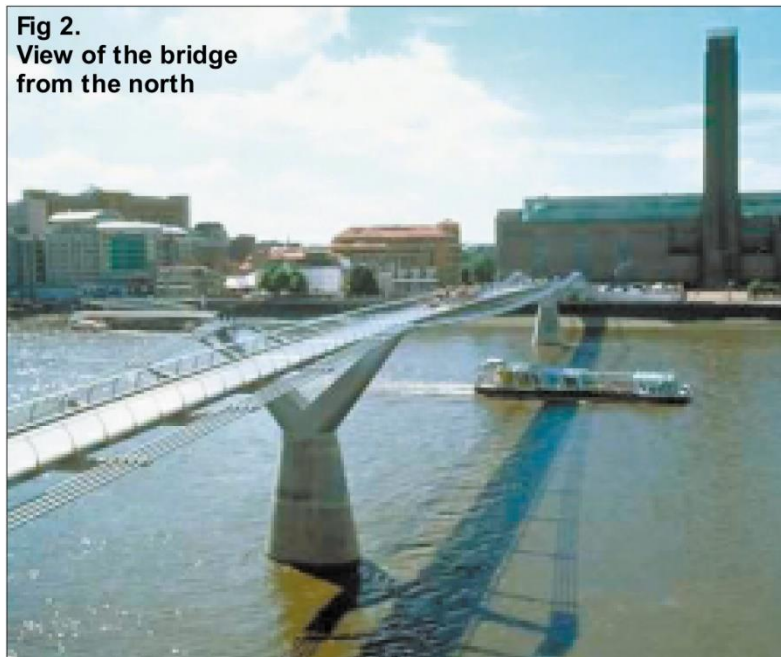


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MILLENNIUM BRIDGE

Em 10 de Junho de 2000, a ponte foi aberta para o público; entre 80 000 e 100 000 pessoas atravessam-na.



MILLENNIUM BRIDGE

Em 10 de Junho de 2000, a ponte foi aberta para o público; entre 80 000 e 100 000 pessoas atravessam-na.



Densidades entre 1.3 e 1.5 $\frac{\text{pessoas}}{\text{m}^2}$

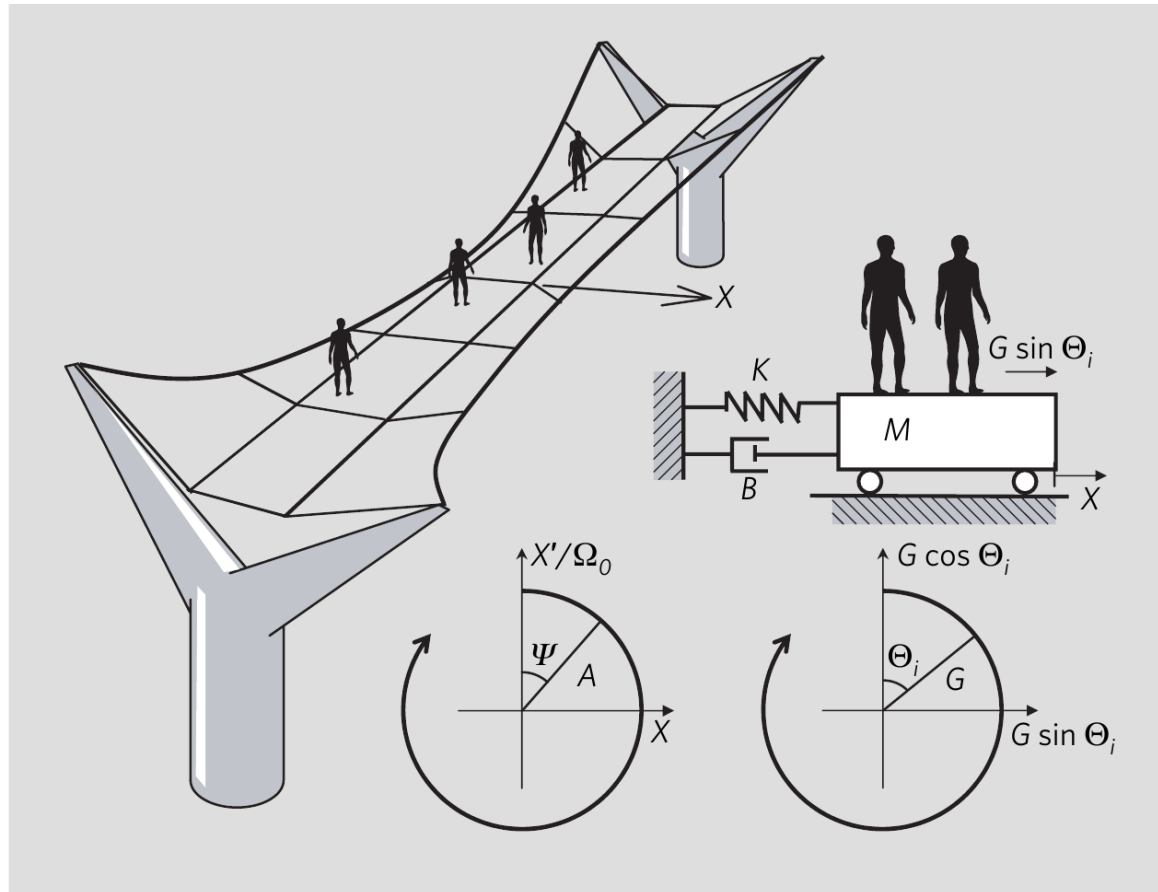
50 a 70 mm de amplitude

www.londonmillenniumbridge.com*

[Dallard, P. et al. *The London Millennium Footbridge* – Struct. Eng. 6 - 2001]



MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA



[Strogatz, S. H., Abrams, D. M., McRobie, A., Eckhardt, B. & Ott, E. **Crowd synchrony on the Millennium Bridge.** *Nature* - 2005]

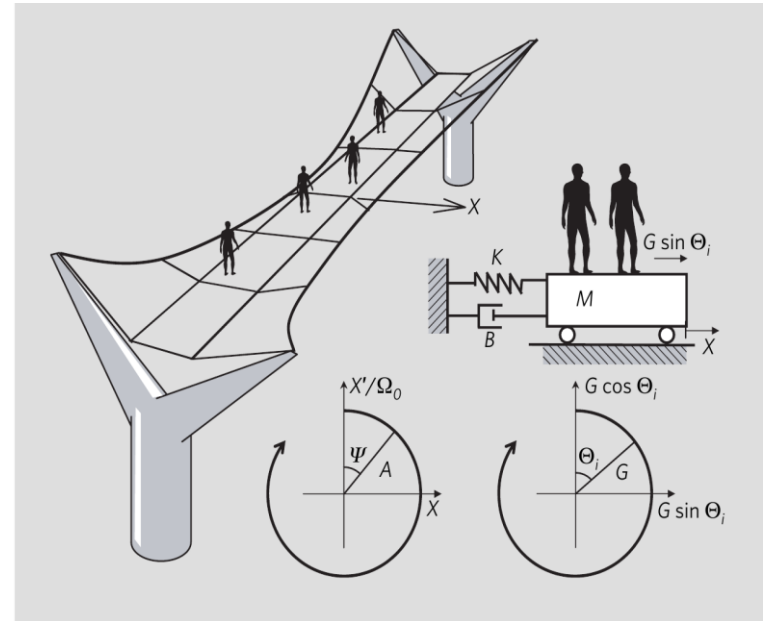
MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

Movimento lateral da ponte

$$M \frac{d^2 X}{dt^2} + B \frac{dX}{dt} + KX = G \sum_{i=1}^N \sin \Theta_i$$

Dinâmica de interação entre a ponte e o pedestre

$$\frac{d\Theta_i}{dt} = \Omega_i + C A \sin(\Psi - \Theta_i + \alpha), \quad i = 1, \dots, N$$



MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

Oscilador harmônico fracamente amortecido e forçado

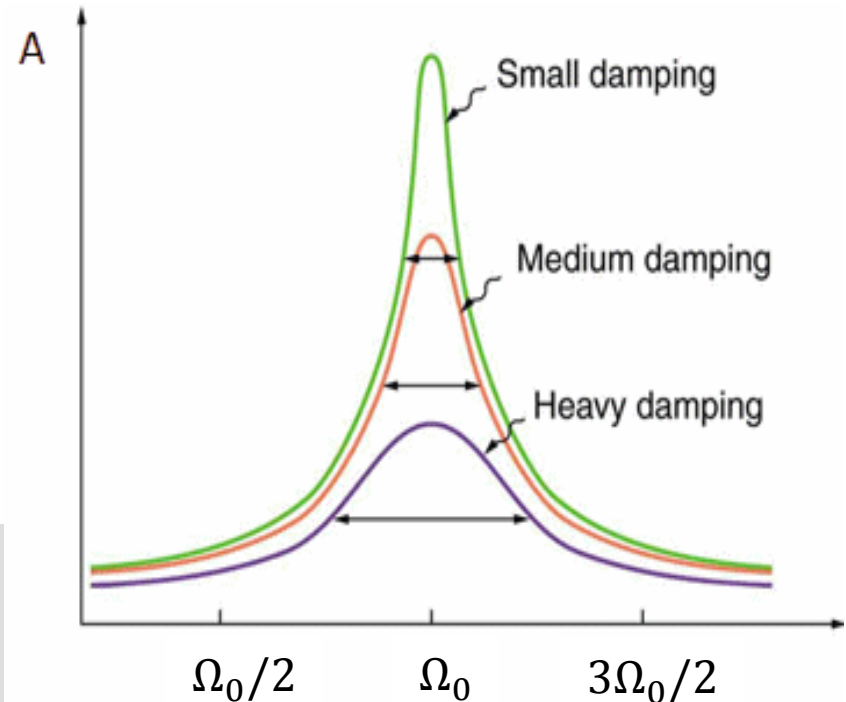
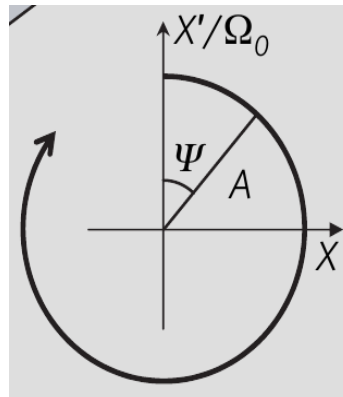
$$M \frac{d^2 X}{dt^2} + B \frac{dX}{dt} + KX = G \sum_{i=1}^N \sin \Theta_i$$

$X(t)$ - deslocamento do *modo lateral*

M - massa modal B - amortecimento

K - rigidez

$$X(t) = A \sin \Psi(t)$$



$$\Omega_0 = \sqrt{K/M}$$

MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

Oscilador harmônico fracamente amortecido e forçado

$$M \frac{d^2 X}{dt^2} + B \frac{dX}{dt} + KX = G \sum_{i=1}^N \sin \Theta_i$$

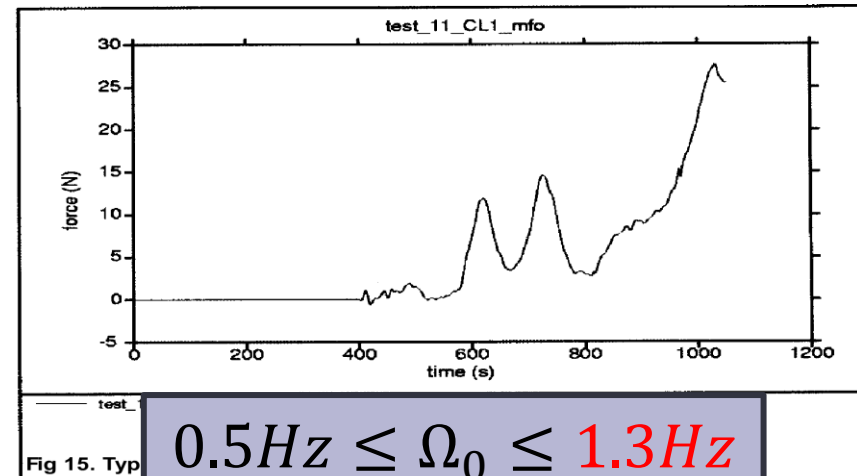
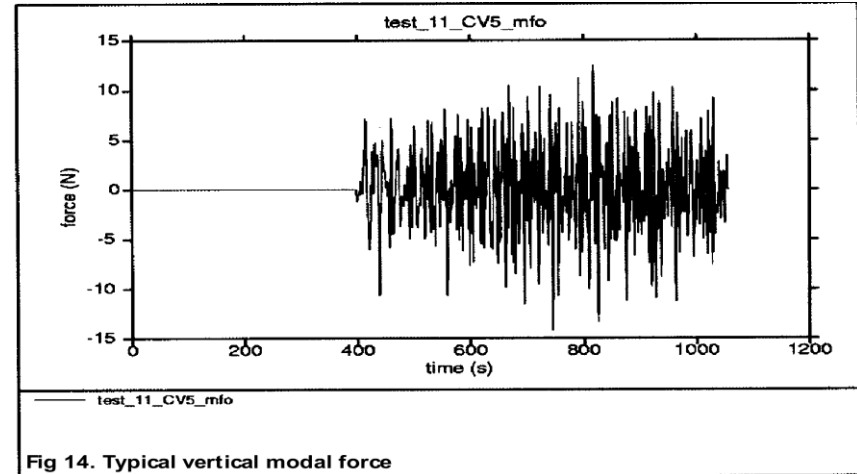
$X(t)$ - deslocamento do *modo lateral*

M - massa modal B - amortecimento

K - rigidez

$$X(t) = A \sin \Psi (t)$$

$$\Omega_0 = \sqrt{K/M}$$



$$0.5\text{Hz} \leq \Omega_0 \leq 1.3\text{Hz}$$

[Dallard, P. et al. *The London Millennium Footbridge* – Struct. Eng. 6 - 2001]

[Strogatz, S. H., Abrams, D. M., McRobie, A., Eckhardt, B. & Ott, E. *Crowd synchrony on the Millennium Bridge*. *Nature* - 2005]

MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

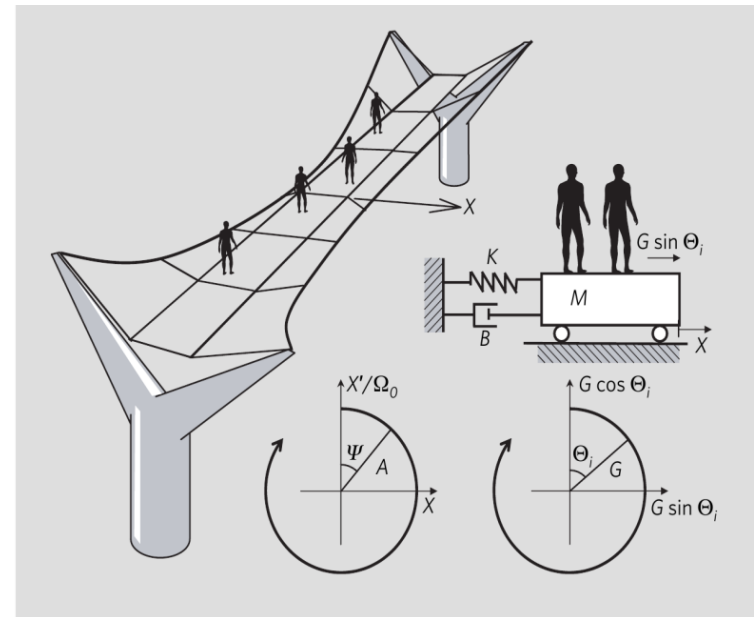
*Dinâmica de interação
entre a ponte e o pedestre*

$$\frac{d\Theta_i}{dt} = \Omega_i + C A \sin(\Psi - \Theta_i + \alpha), \quad i = 1, \dots, N$$

C – sensibilidade do pedestre

α – fase de atraso

Ω_i - frequência distribuída aleatoriamente por uma densidade $P(\Omega)$



MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

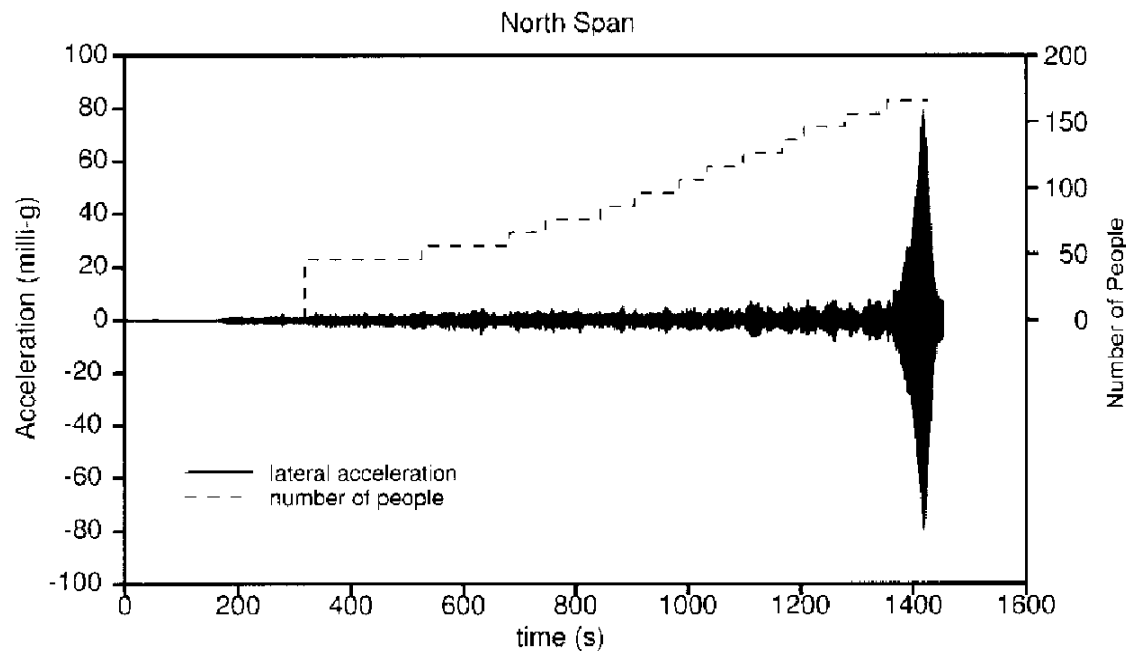


FIG. 4. Lateral Acceleration of Bridge Deck and Number of Pedestrians

Como determinar o número de pessoas N_c ?

[Dallard, P. et al. **LONDON MILLENNIUM BRIDGE: PEDESTRIAN-INDUCED LATERAL VIBRATION** - J. Bridge Eng. 6 - 2001]

[Strogatz, S. H., Abrams, D. M., McRobie, A., Eckhardt, B. & Ott, E. **Crowd synchrony on the Millennium Bridge.** *Nature* - 2005]



MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

Como determinar o número de pessoas N_c ?

- 1) Tirar a dimensão das equações;
- 2) Impor uma teoria de médias para o problema;
- 3) *Estudar as soluções estacionárias**;

$$N_c = \frac{4\beta}{\pi} \left(\frac{K}{GCP(\Omega_0)} \right)$$

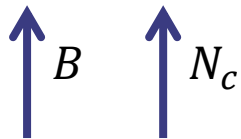
$$\beta = \frac{B}{\sqrt{4MK}}$$

Solução para não destruir a ponte?



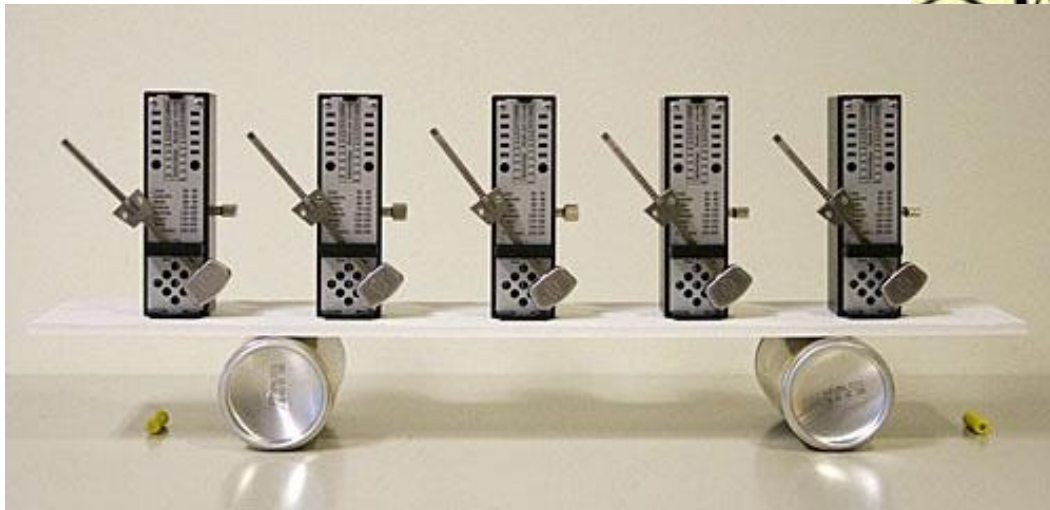
MILLENNIUM BRIDGE – DESCRIÇÃO DO PROBLEMA

$$N_c = \frac{4\beta}{\pi} \left(\frac{K}{GCP(\Omega_0)} \right) \quad \beta = \frac{B}{\sqrt{4MK}}$$



- $2 \frac{\text{pessoas}}{\text{m}^2}$
- 18% amortecimento dos modos laterais abaixo de 1.3Hz

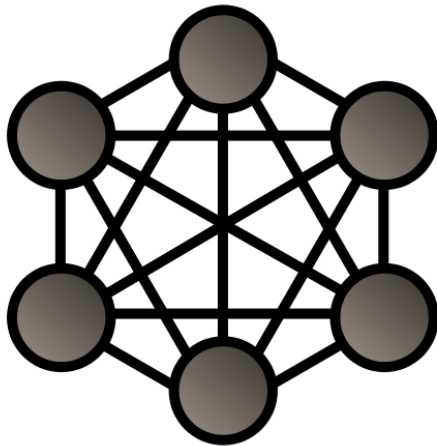




MODELO DE KURAMOTO

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i), \quad i = 1, \dots, N$$

(1975)



Yoshiki Kuramoto

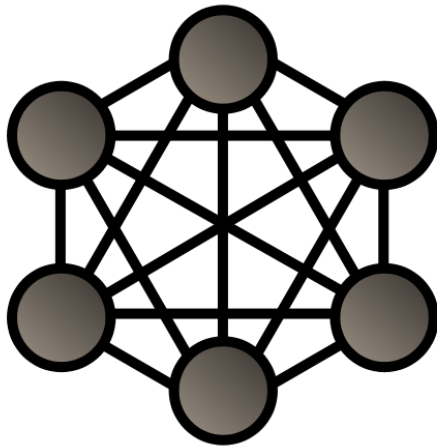
MODELO DE KURAMOTO

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i), \quad i = 1, \dots, N$$

Termo individual

Termo de interação – Acoplamento fraco

(1975)



Yoshiki Kuramoto

MODELO DE KURAMOTO

$$\dot{\theta}_i = \underbrace{\omega_i}_{\text{Termo individual}} + \underbrace{\frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)}_{\text{Termo de interação - Acoplamento fraco}}, \quad i = 1, \dots, N$$

Termo individual

Termo de interação - Acoplamento fraco

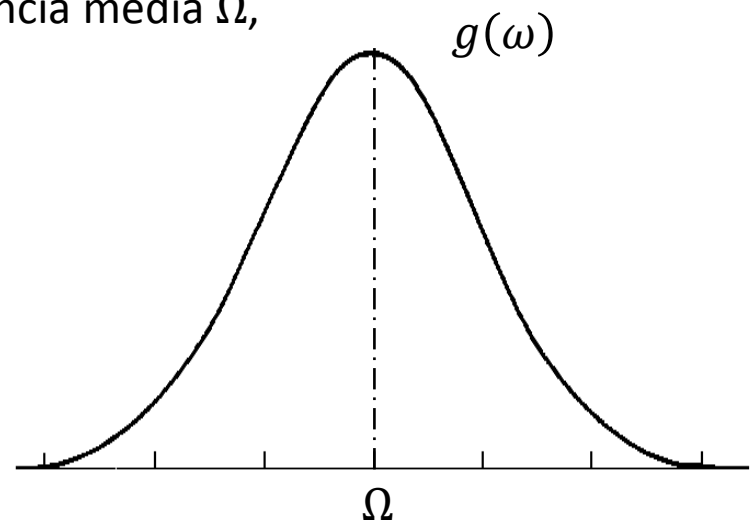
ω_i - frequência natural distribuído por uma densidade de probabilidade $g(\omega)$, onde

$g(\omega)$ é unimodal e simétrica em relação à frequência média Ω ,

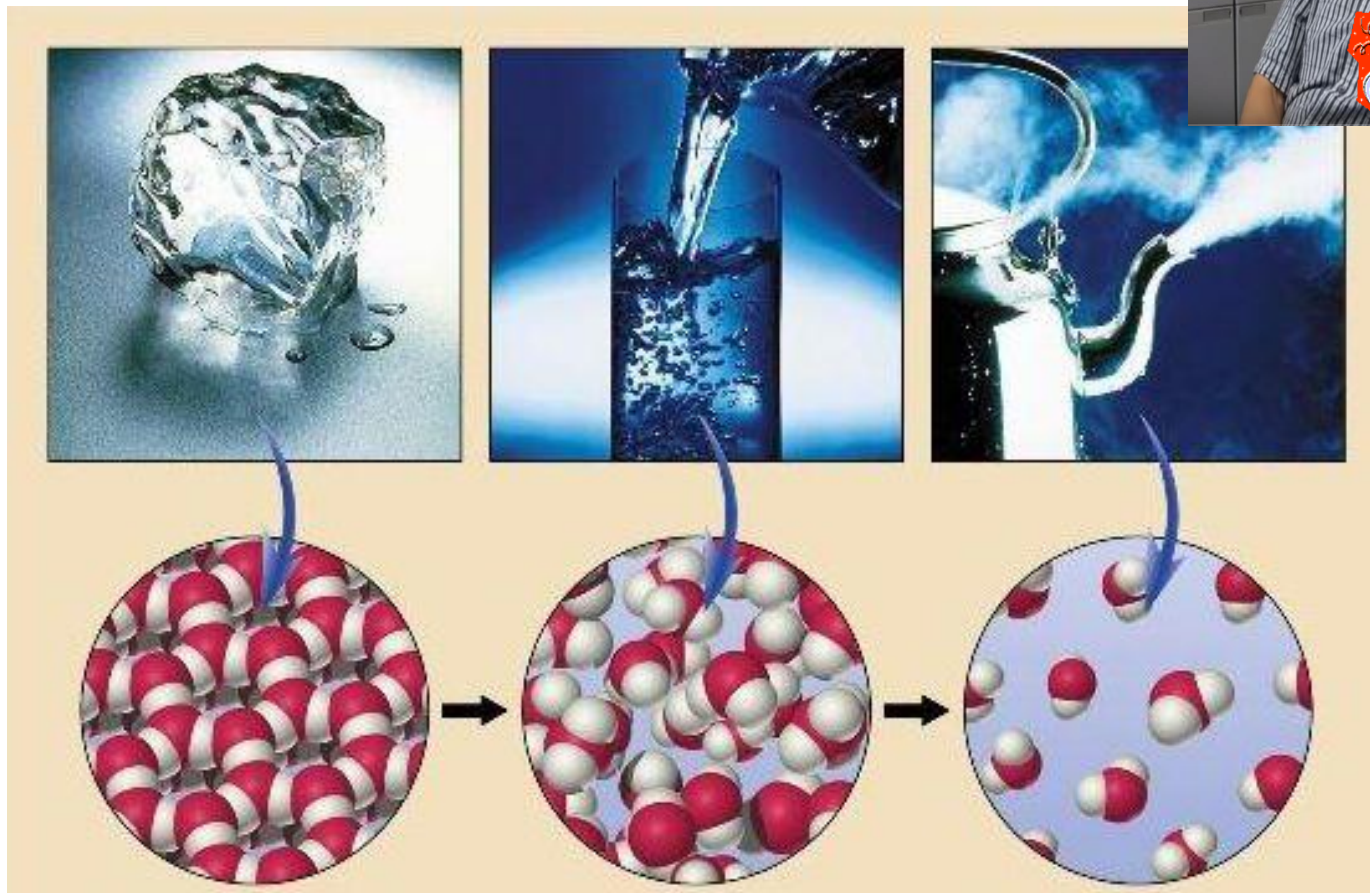
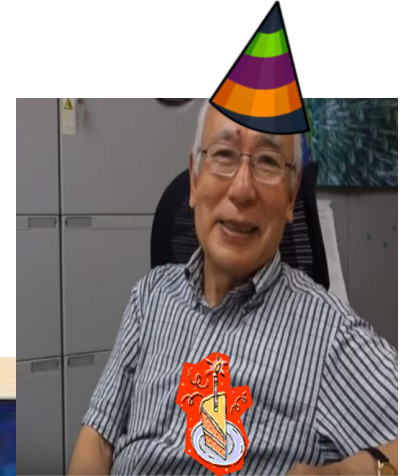
$g(\Omega + \omega) = g(\Omega - \omega)$.

K - força de acoplamento

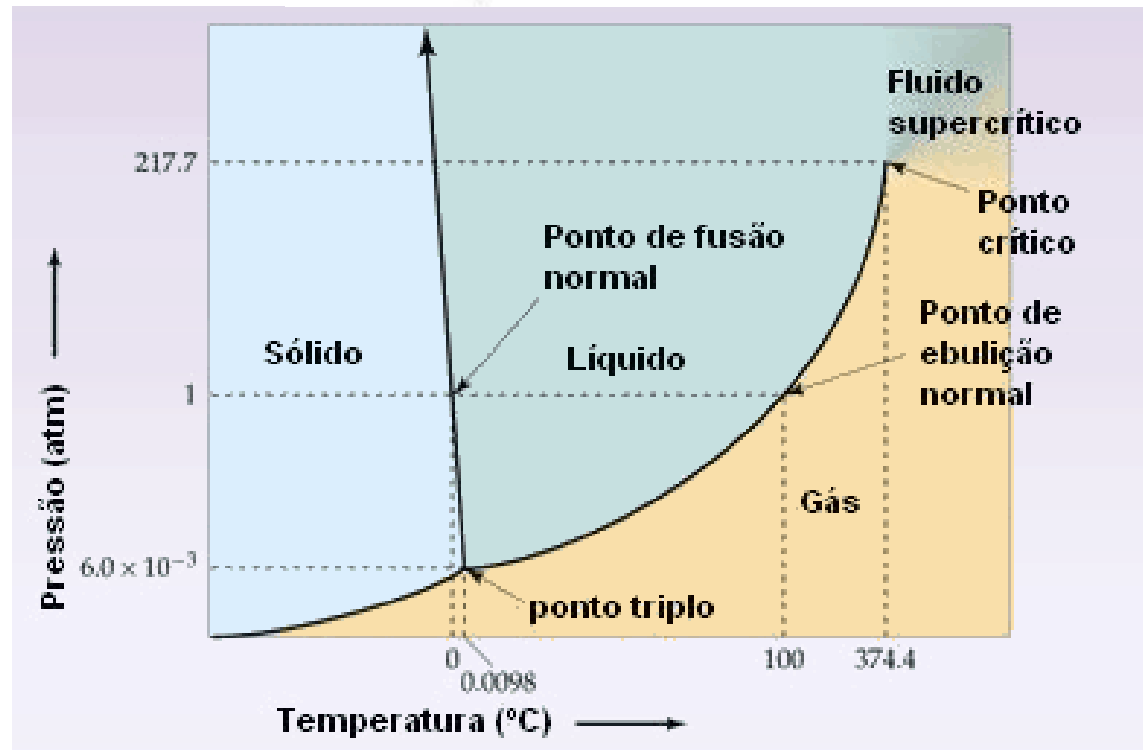
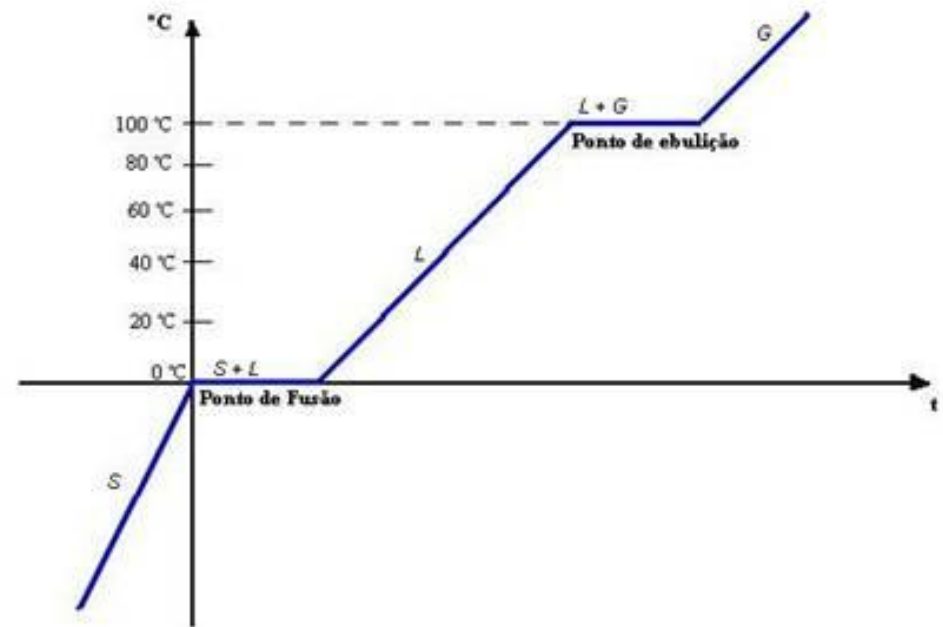
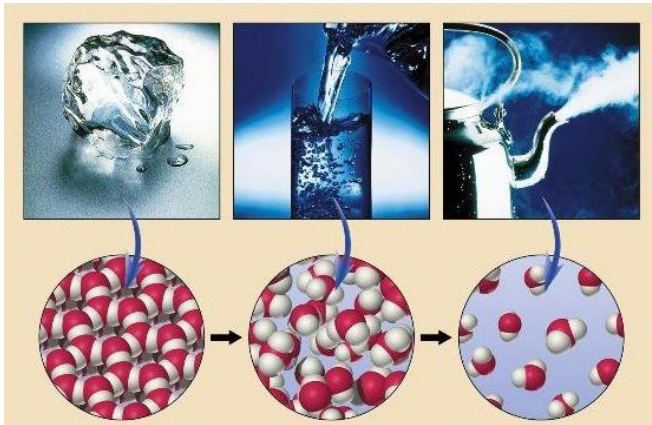
N - número de osciladores



TRANSIÇÃO DE FASE

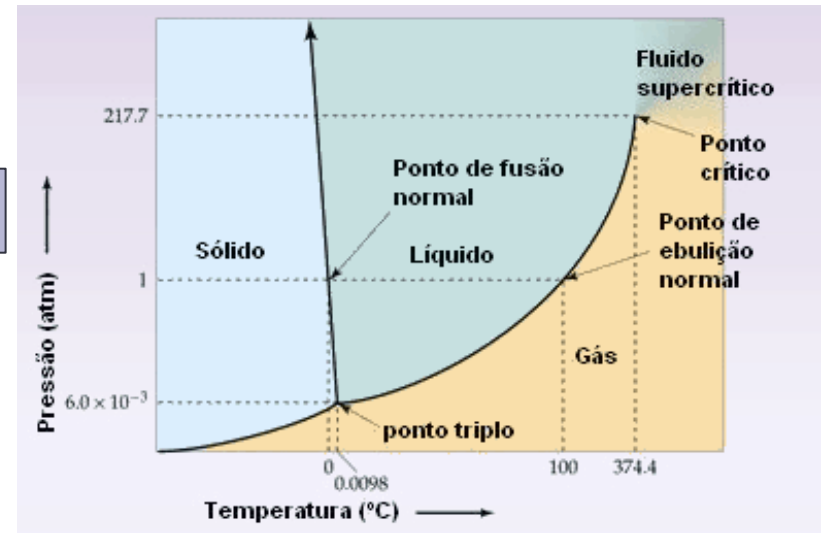
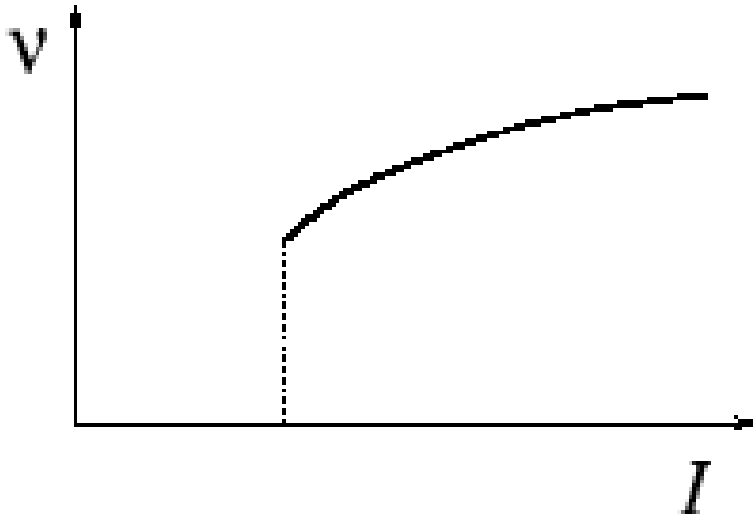


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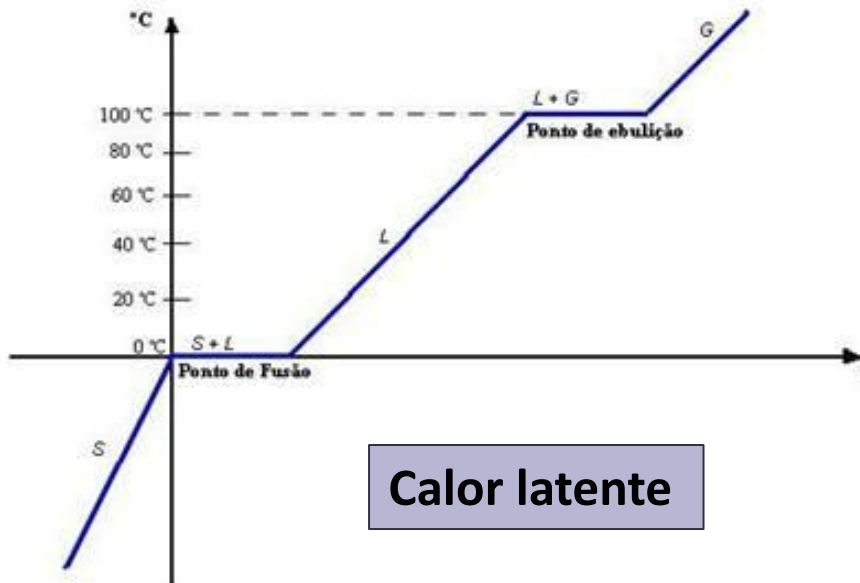
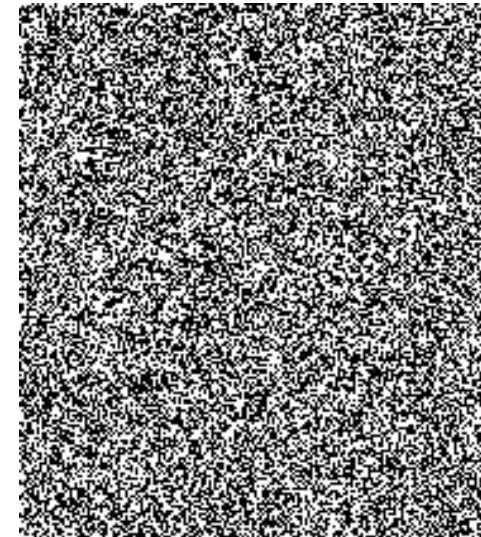


TRANSIÇÃO DE FASE

Transição de fase descontínua (1ª ordem)



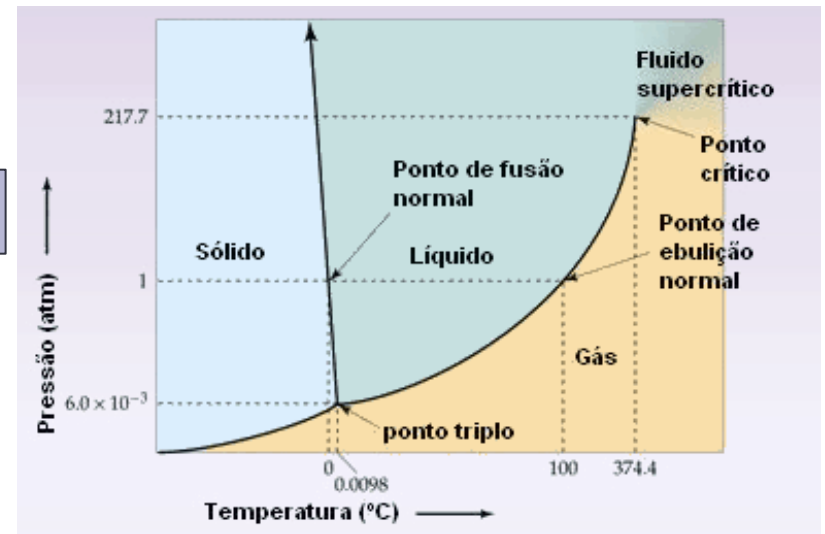
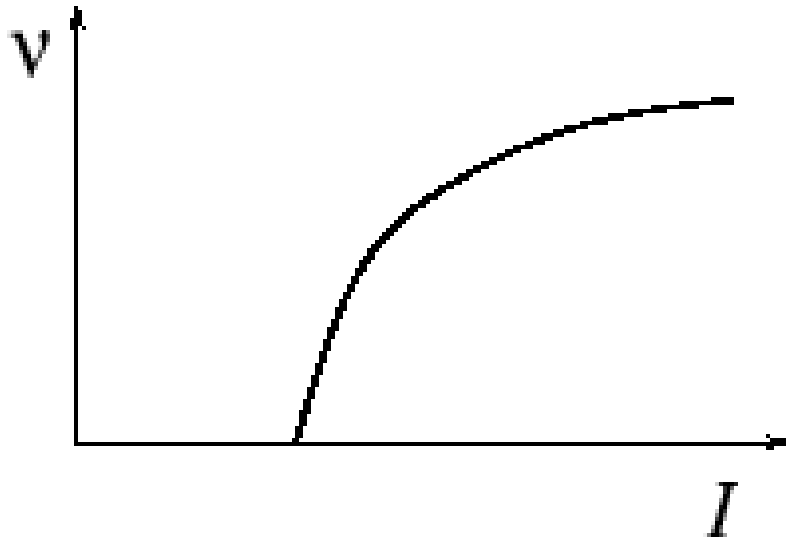
Coexistência de fases



Calor latente

TRANSIÇÃO DE FASE

Transição de fase contínua (2ª ordem)



Conglomerados



MODELO DE KURAMOTO

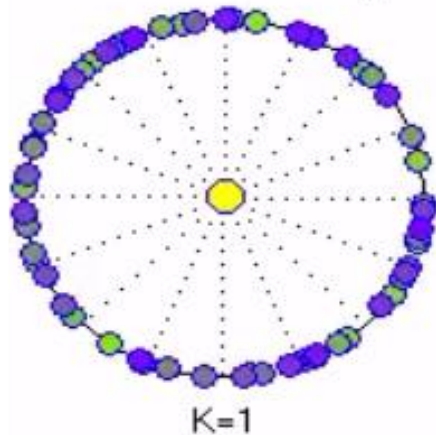
Parâmetro de ordem

$$r(t) e^{i\psi(t)} = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j(t)}$$

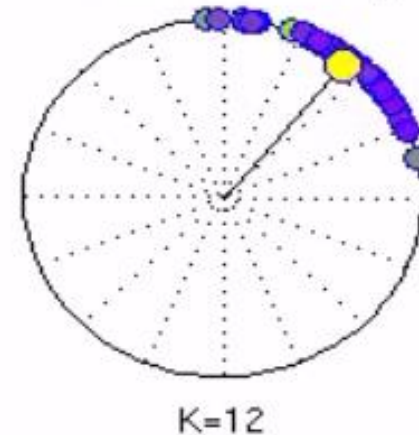
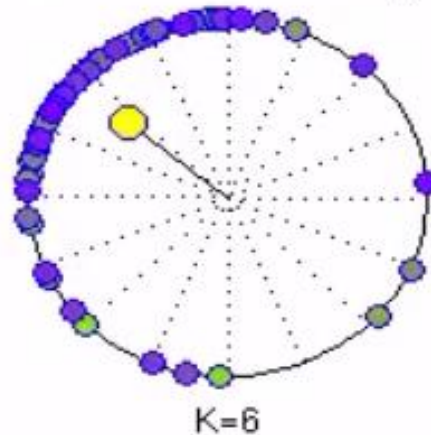
$$0 \leq r(t) \leq 1$$

$\psi(t)$ – fase média

Assíncrono



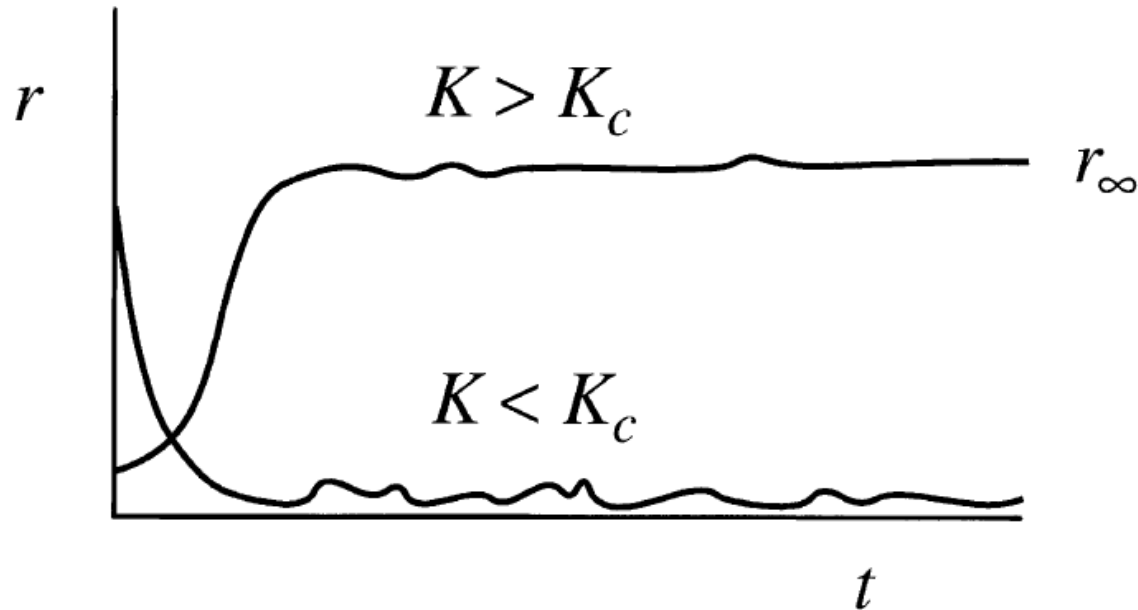
Síncrono



Synchronization phase transition



MODELO DE KURAMOTO



$$K_c = ?$$

[Strogatz, S.H. *From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators*. Physica D – 2000.]

[Kuramoto, Y. *Chemical Oscillations, Waves and Turbulence* - Springer-Verlag 1984.]



MODELO DE KURAMOTO

$$K_c = ?$$

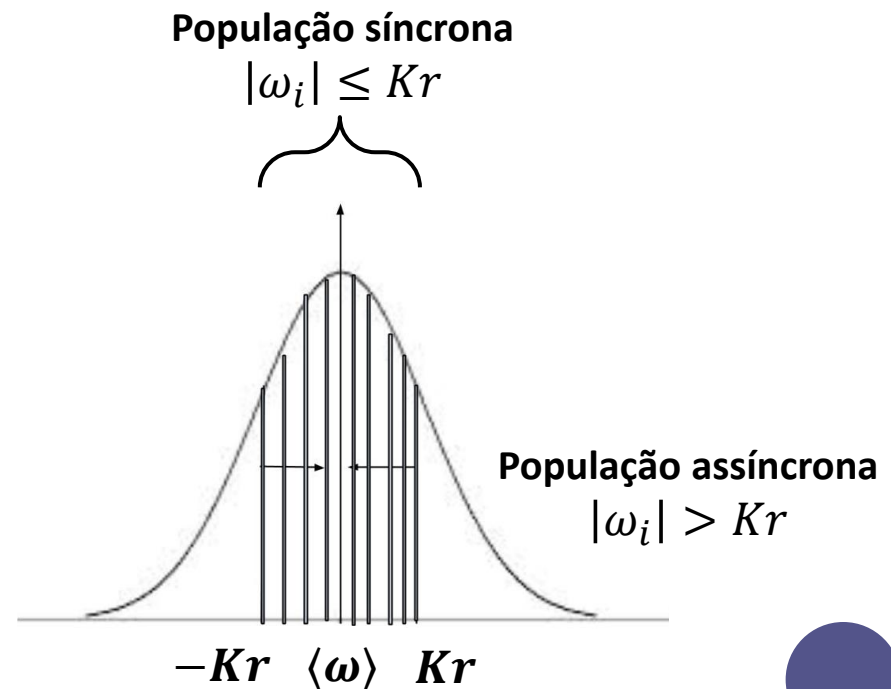
Limite do contínuo:

$$N \rightarrow \infty$$

$\rho(\theta, \omega)d\theta$ – densidade de osciladores com frequência ω entre θ e $\theta + d\theta$

3) Estudar as soluções estacionárias*;

$$r = \langle e^{i\theta} \rangle_{\text{síncrono}} + \langle e^{i\theta} \rangle_{\text{assíncrono}}$$



MODELO DE KURAMOTO

$$K_c = ?$$

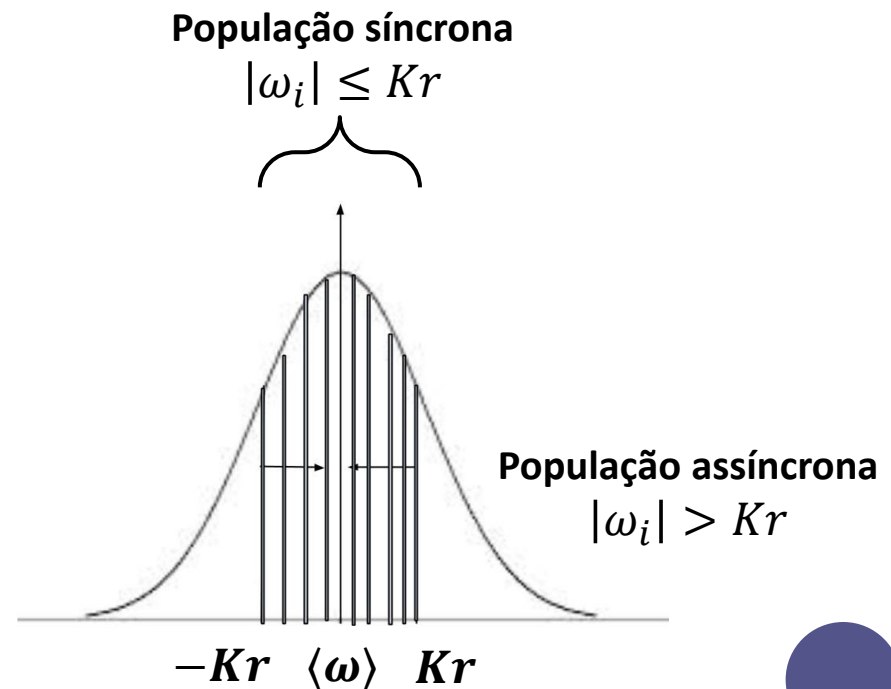
Limite do contínuo:

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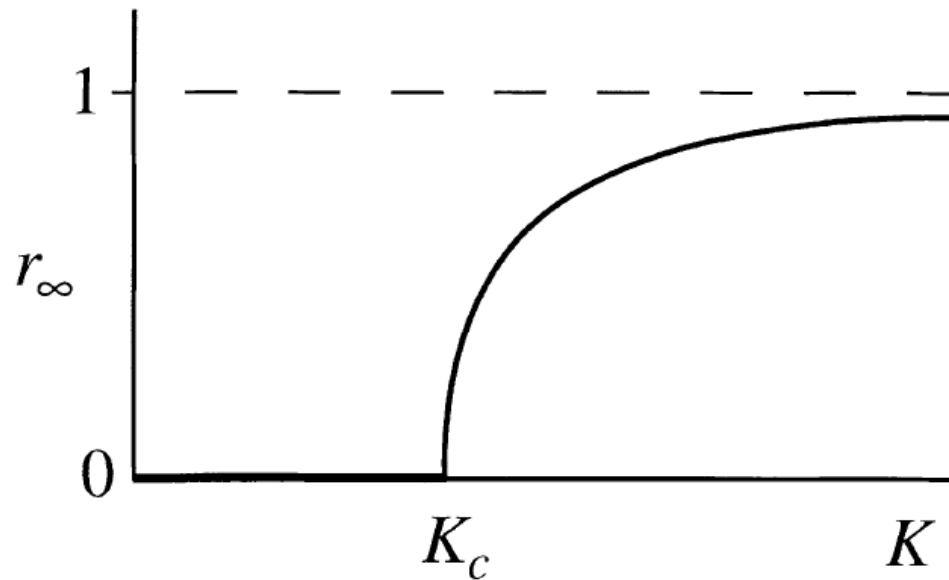
$\rho(\theta, \omega)d\theta$ – densidade de osciladores com frequência ω entre θ e $\theta + d\theta$

3) Estudar as soluções estacionárias*;

$$r = \langle e^{i\theta} \rangle_{\text{síncrono}} + \langle e^{i\theta} \rangle_{\text{assíncrono}} \rightarrow 0$$



MODELO DE KURAMOTO



$$K_c = \frac{2}{\pi g(0)}$$

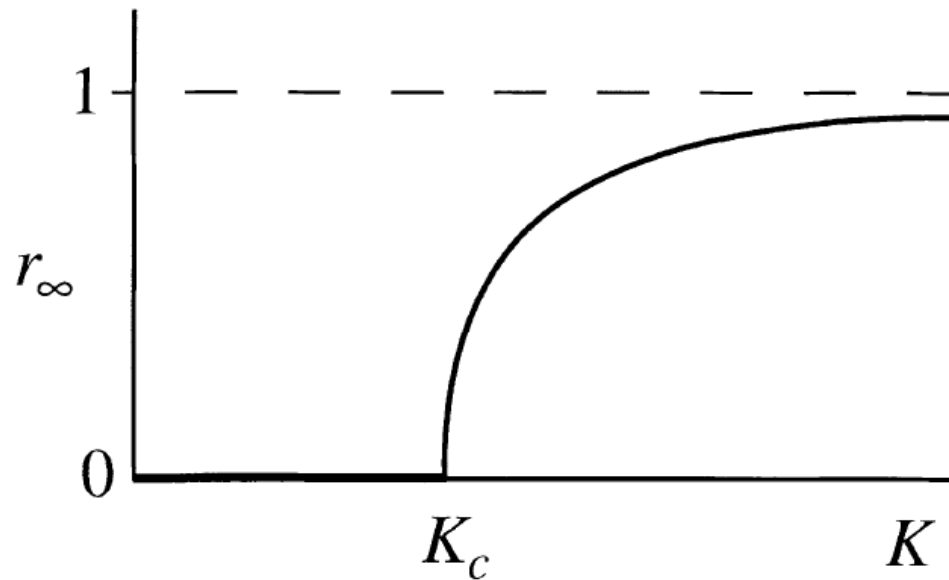
Synchronization phase transition

Transição de fase contínua

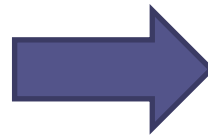
[Strogatz, S.H. *From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators*. Physica D – 2000.]



MODELO DE KURAMOTO



$$K_c = \frac{2}{\pi g(0)}$$



$$N_c = \frac{4\beta}{\pi} \left(\frac{K}{GCP(\Omega_0)} \right)$$

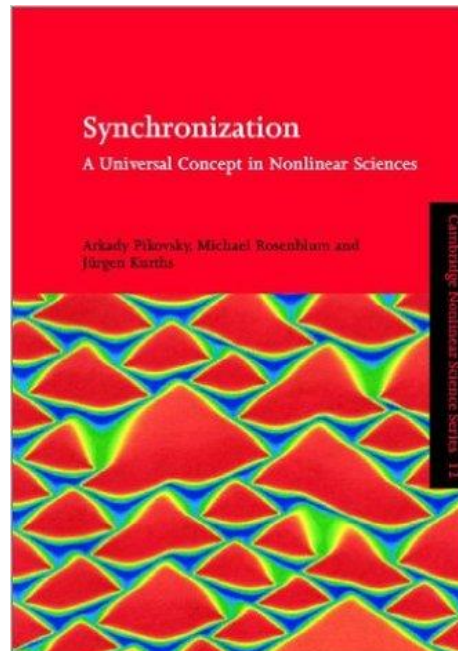
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Mais informações:

The screenshot shows the TED website interface for a video. At the top left is the TED logo, followed by navigation links: Watch, Discover, Attend, Participate, and About. On the top right, there is a search bar with the text "Search...", and links for "Log in" and "Sign up". The main content area features a video player with a dark background. On the left side of the player, the text reads "Steven Strogatz:" followed by the title "Steven Strogatz fala sobre sincronização" in large white font. Below the title, it says "TED2004 · 21:58 · Filmed Feb 2004" and "Subtitles available in 19 languages". At the bottom left of the player, there is a "View interactive transcript" link and a "Play" button. A large white play button icon is centered over the video. On the right side of the player, there is a vertical menu of icons: a clock for "Watch later", a heart for "Favorite", a download arrow for "Download", and a three-dot menu for "Rate". The background of the video player shows a man (Steven Strogatz) in a dark shirt, looking towards the camera.



Agradecimentos:



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OBRIGADO!

