

A instabilidade da geometria euclidiana

Seminário de coisas legais

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Universidade de São Paulo

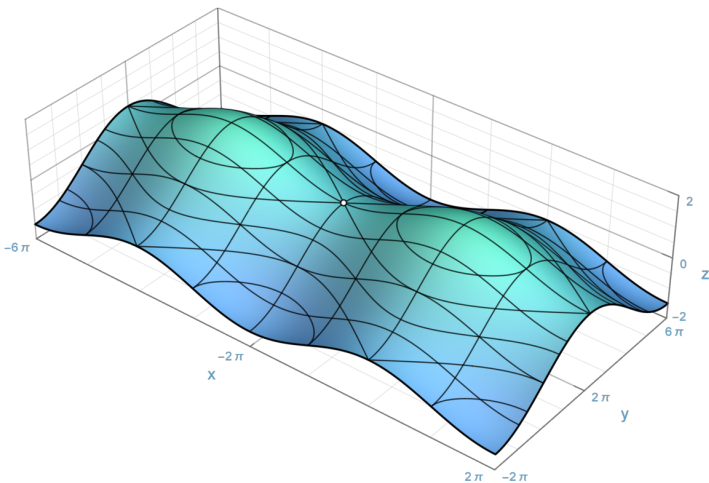
Novembro, 2022



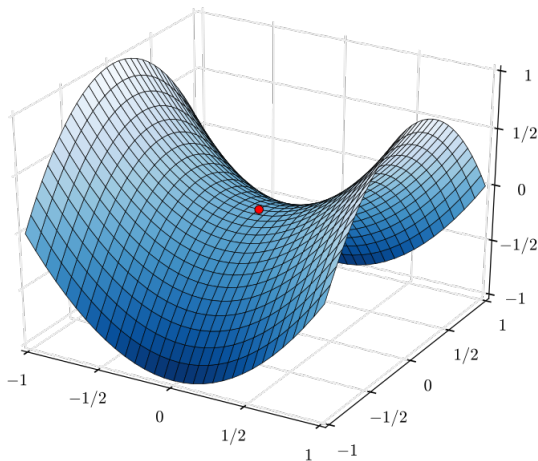
- ① Curvatura
- ② Espaço das retas orientadas
- ③ Espaço dos triângulos
- ④ Referências

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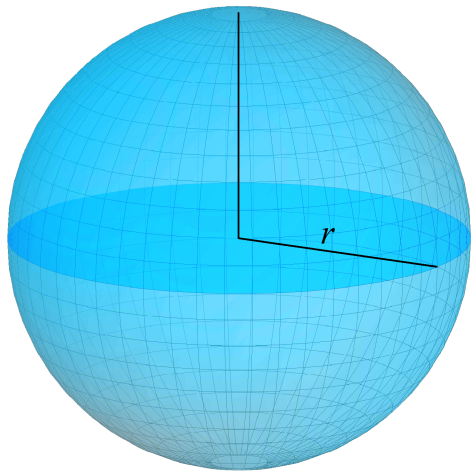
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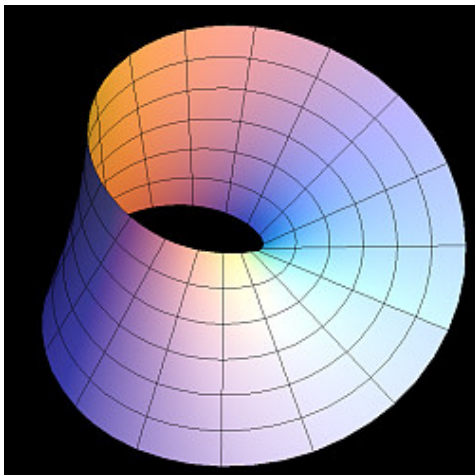
Superfícies



Superfícies

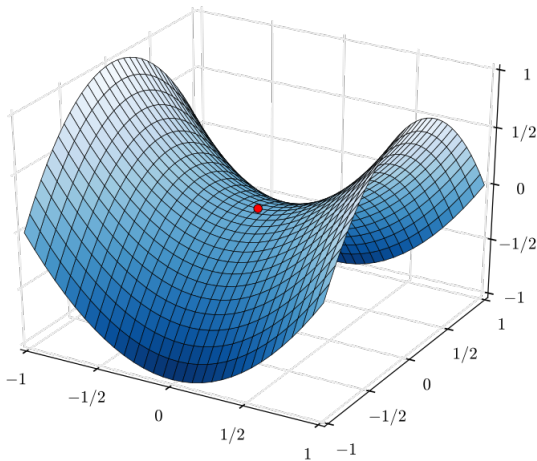


Superfícies



Curvatura

$K < 0$:



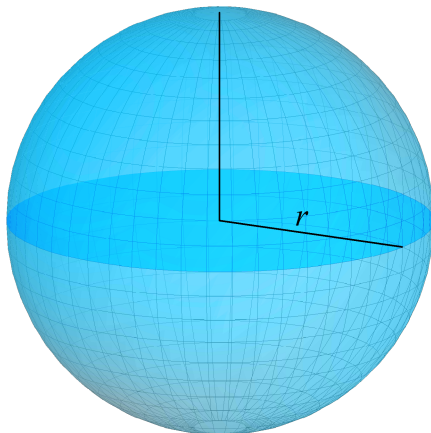
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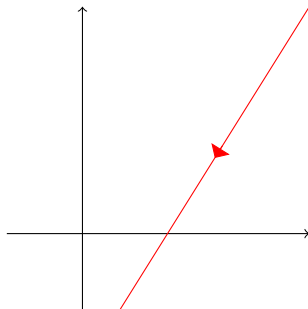
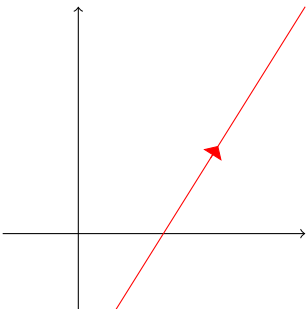
Curvatura

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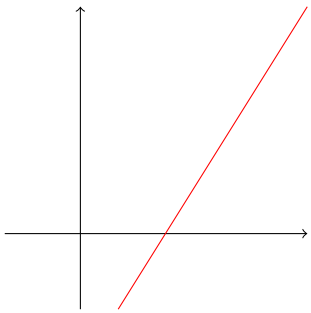


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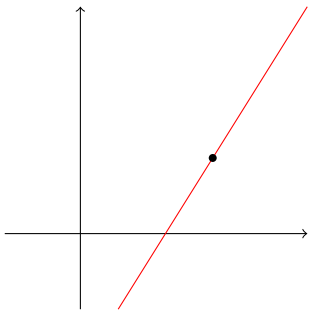
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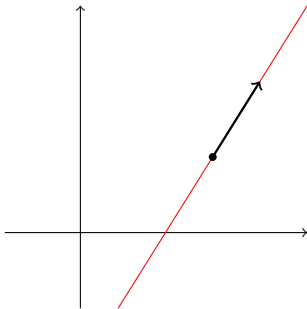
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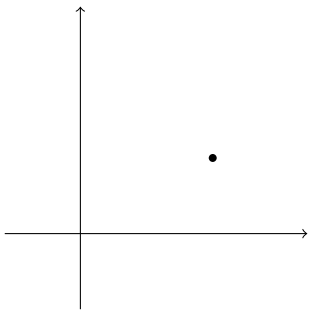
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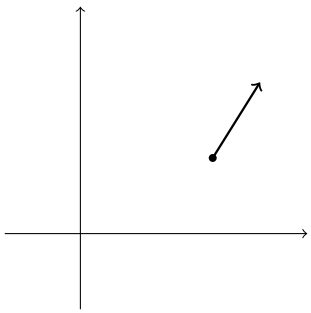
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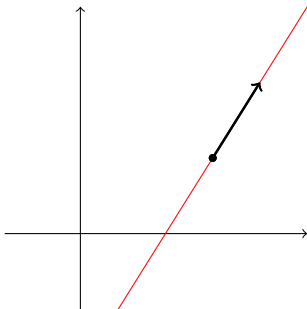
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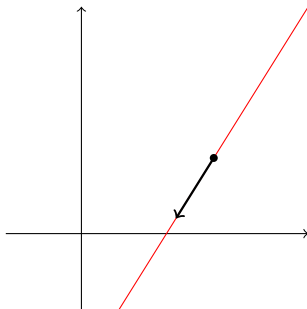
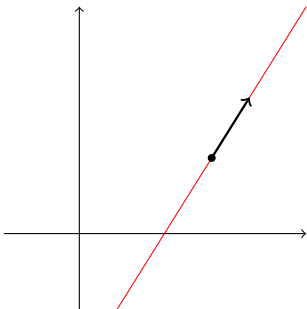
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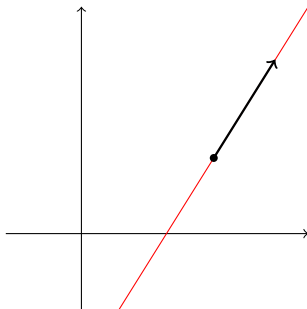
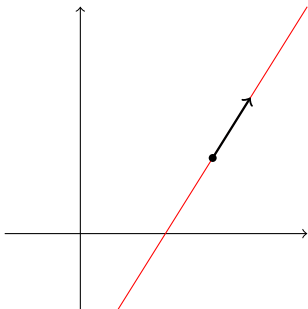
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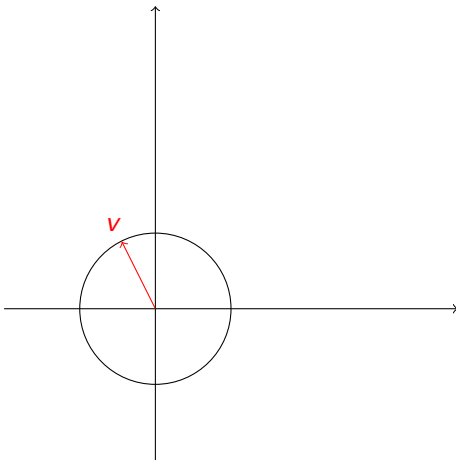
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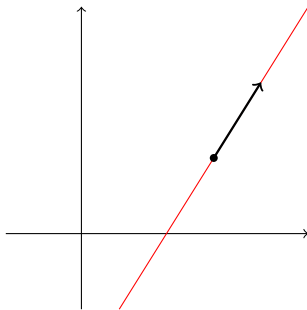
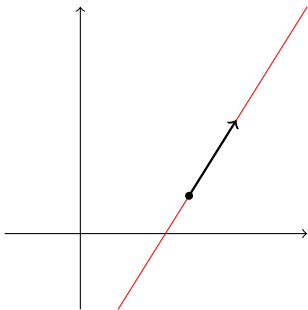
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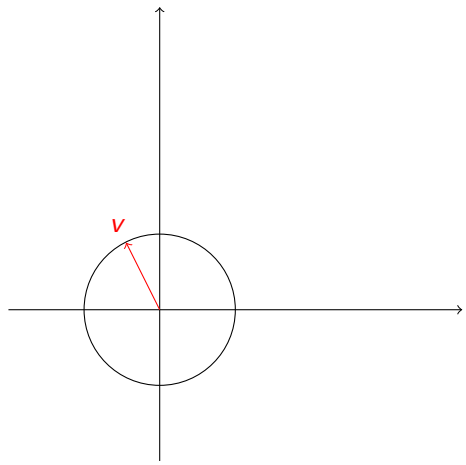
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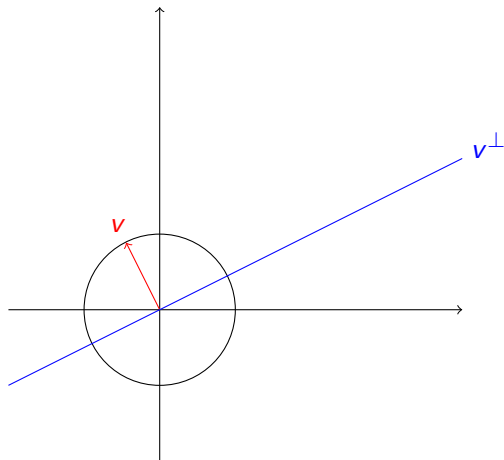
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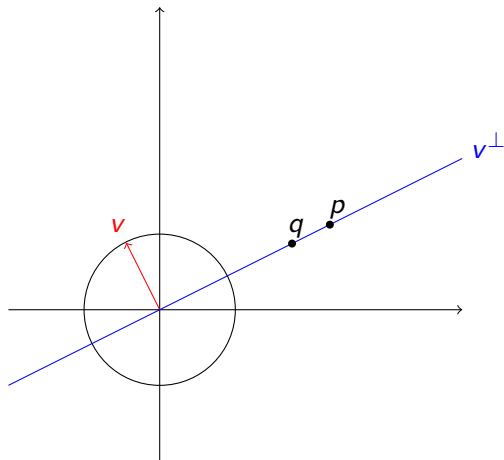
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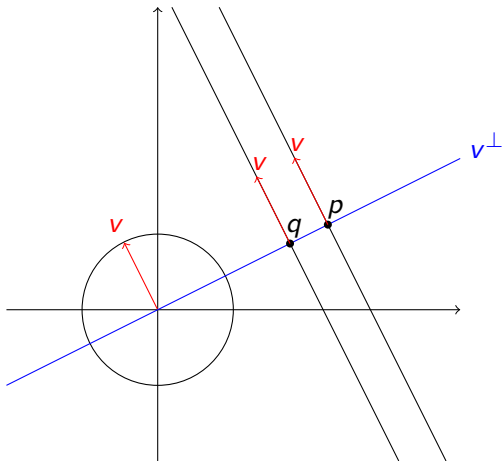
Tirando redundâncias



Tirando redundâncias



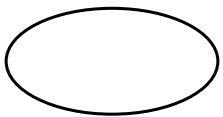
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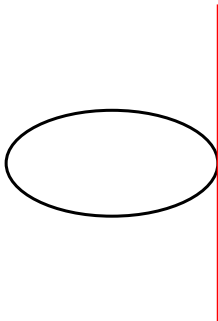
Conclusão

Retas \longleftrightarrow pares (v, p) , onde v é um vetor que está no círculo de raio 1, e p é um ponto que está na reta perpendicular a v .

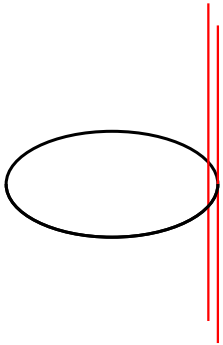
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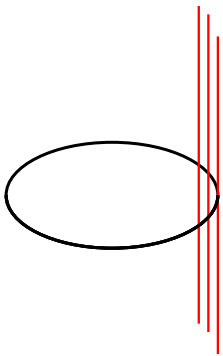
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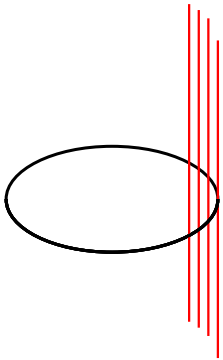
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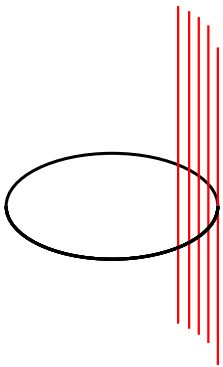
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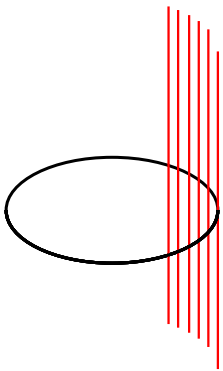
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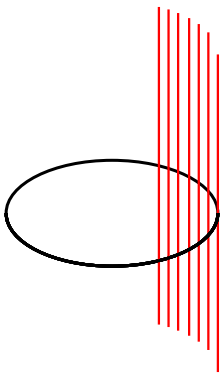
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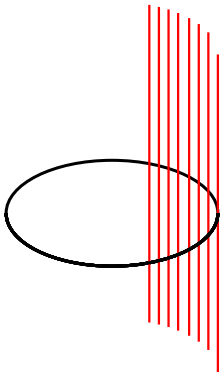
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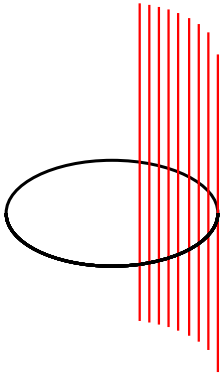
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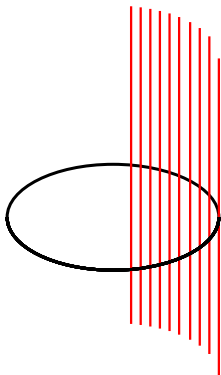
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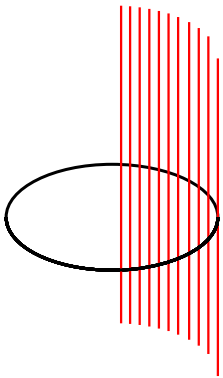
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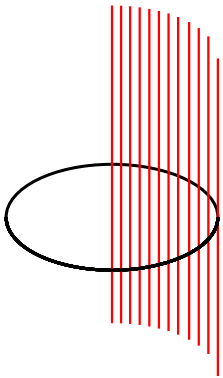
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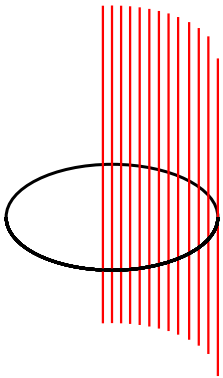
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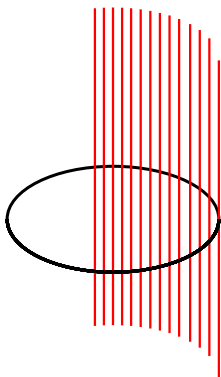
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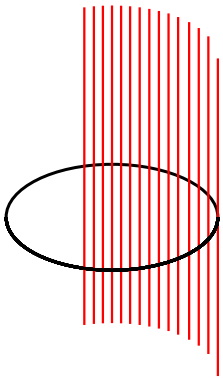
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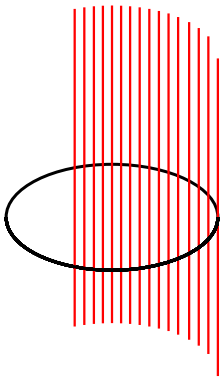
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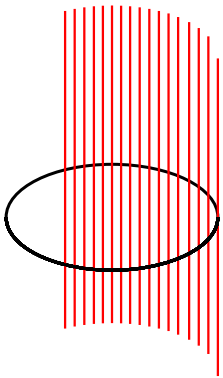
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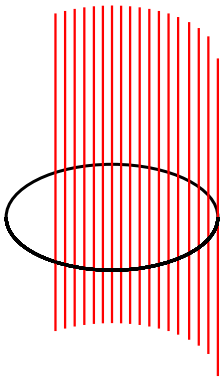
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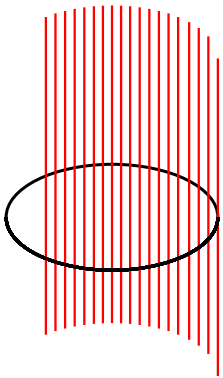
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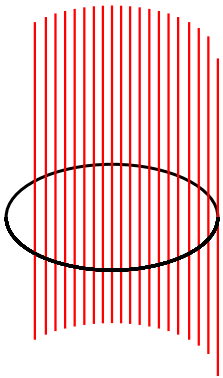
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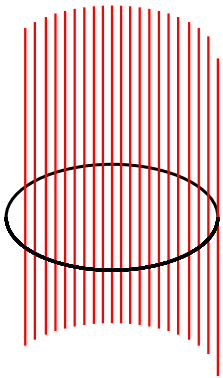
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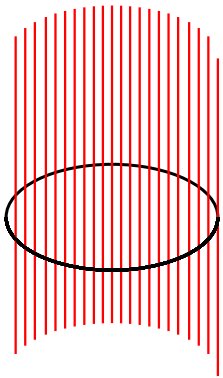
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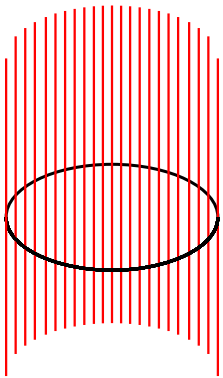
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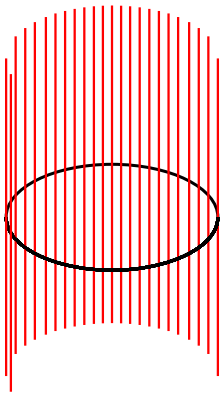
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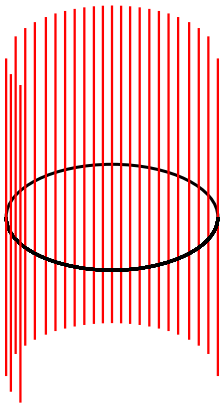
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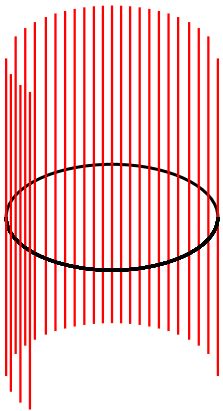
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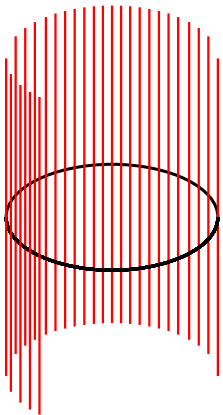
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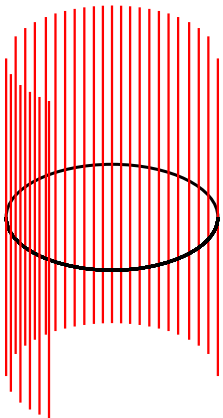
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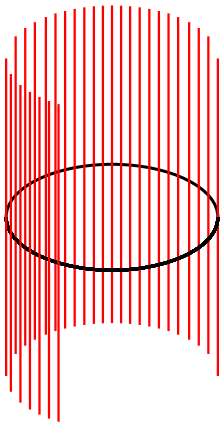
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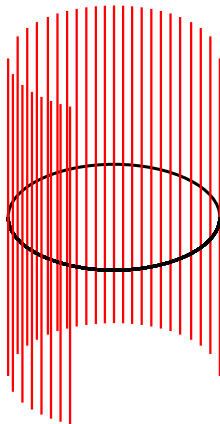
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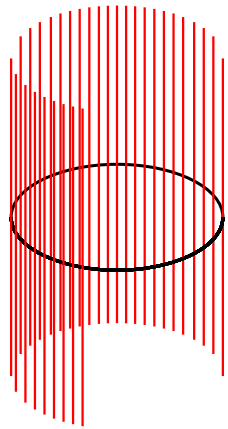
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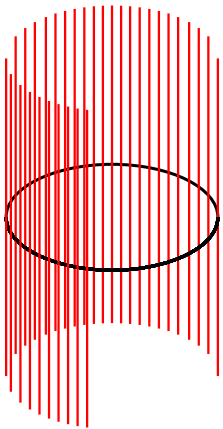
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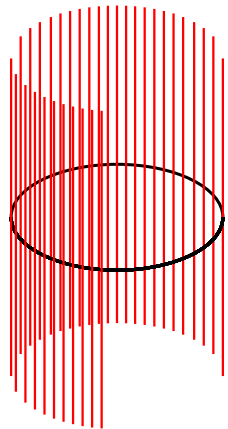
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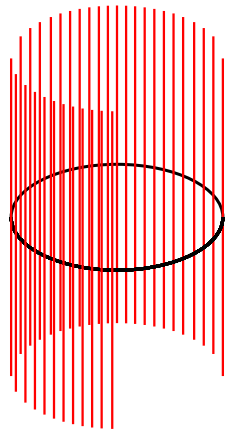
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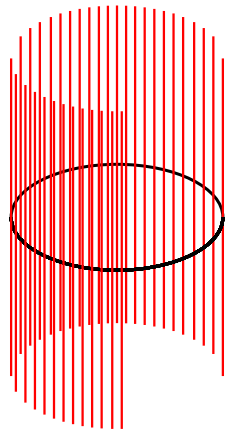
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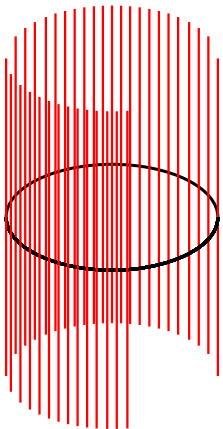
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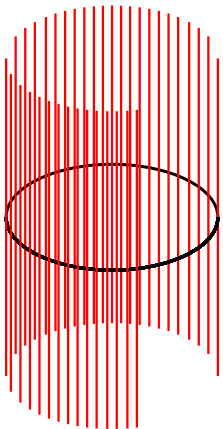
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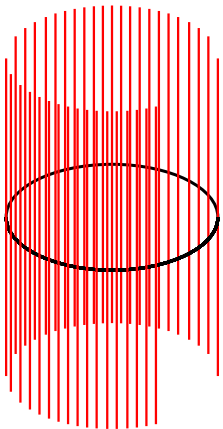
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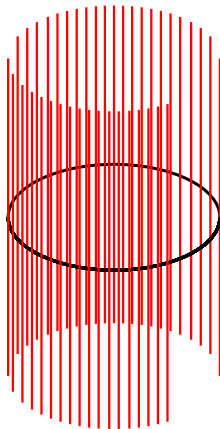
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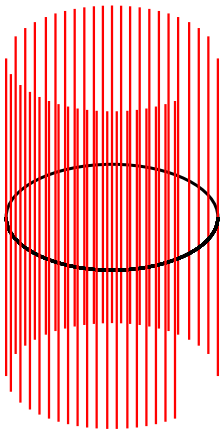
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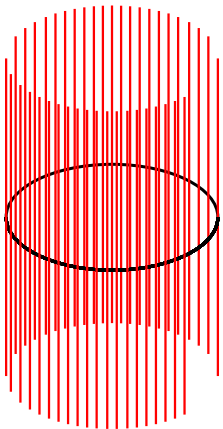
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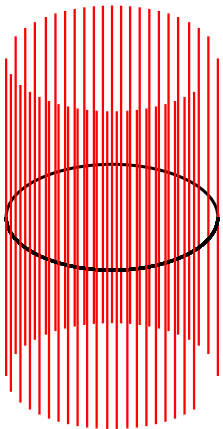
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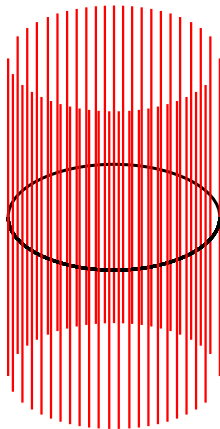
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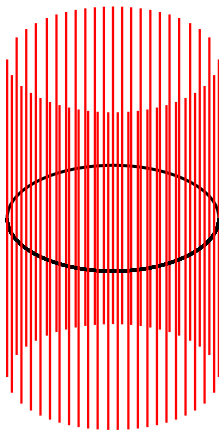
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Conclusão



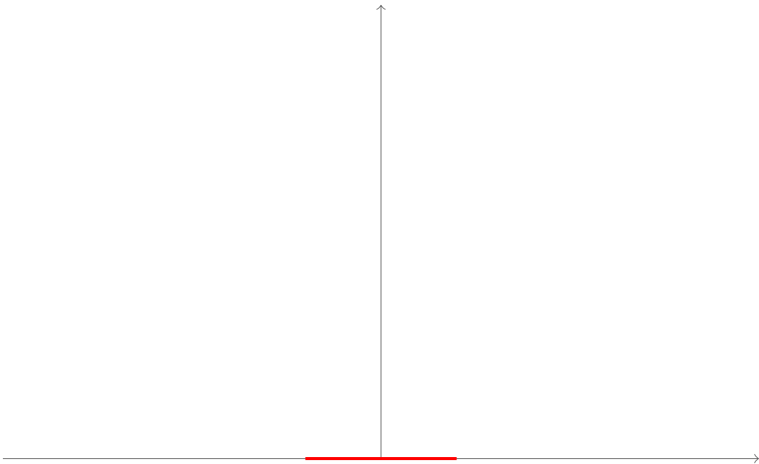
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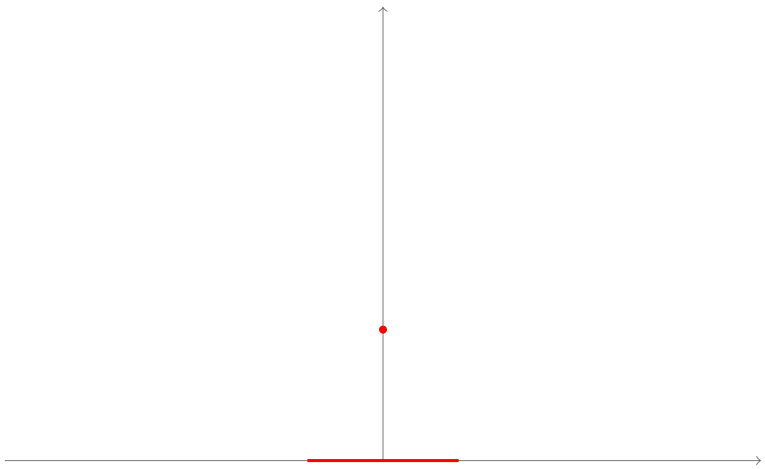
CILINDRO!

- 1 Curvatura
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- 3 Espaço dos triângulos**
- 4 Referências

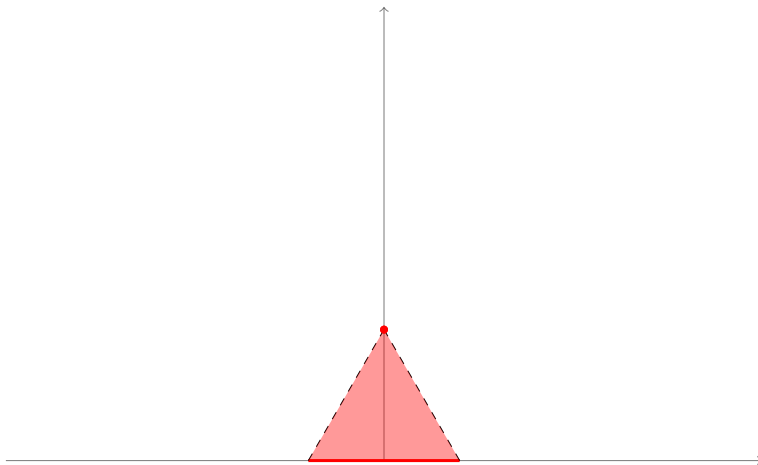
O semiplano



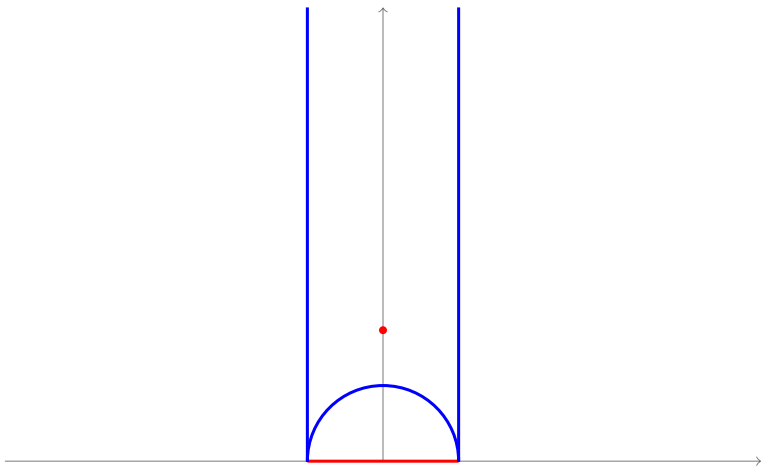
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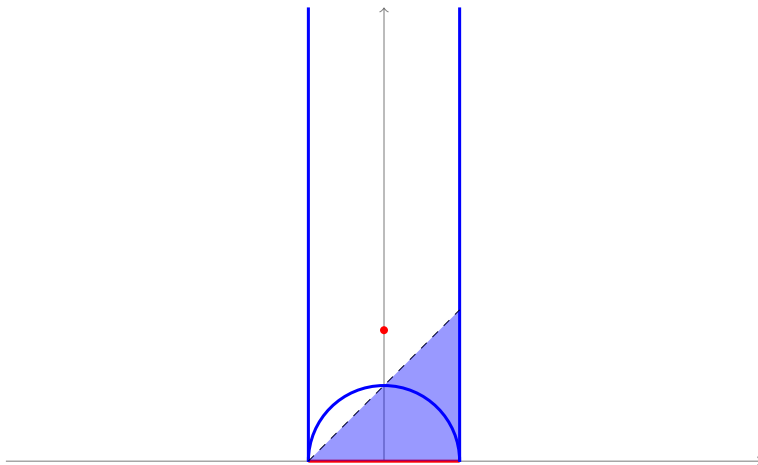
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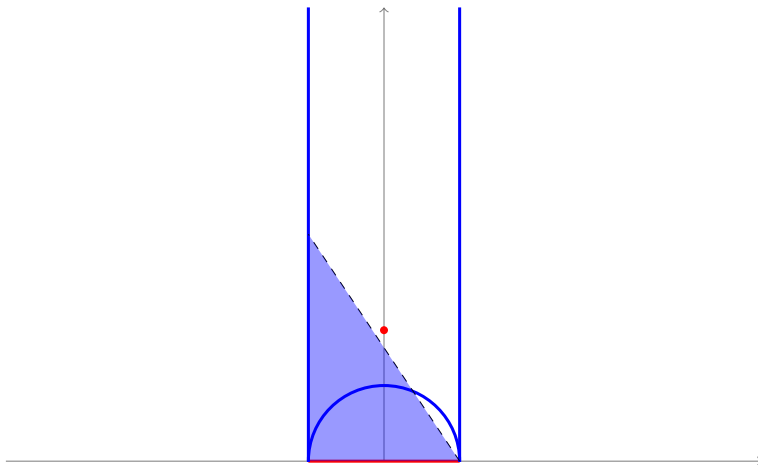
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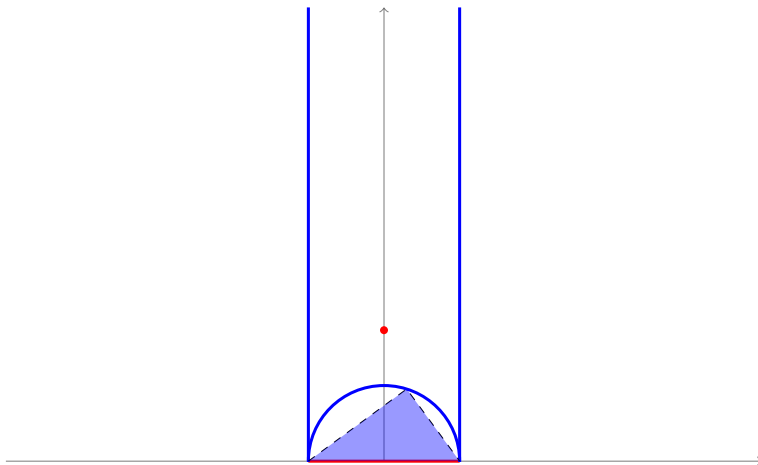
Retângulos



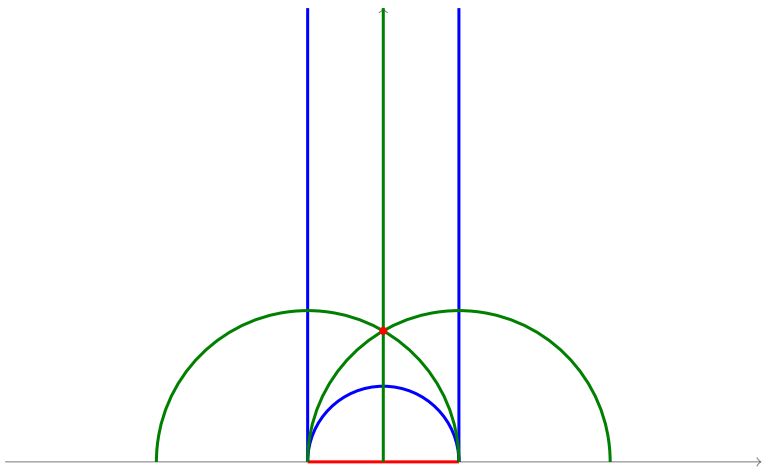
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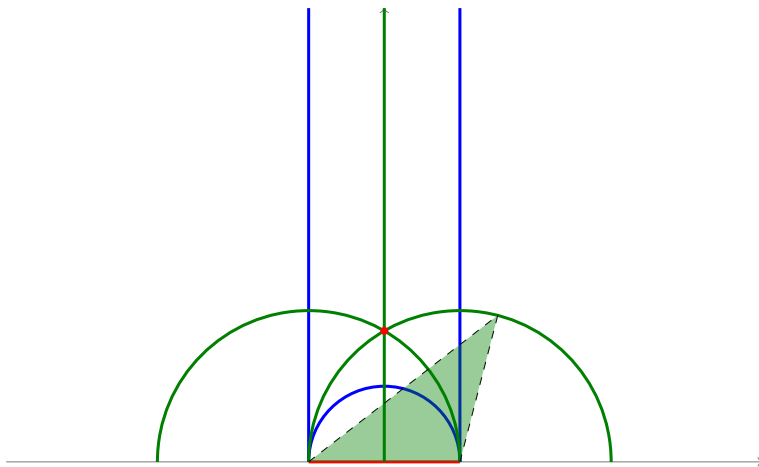
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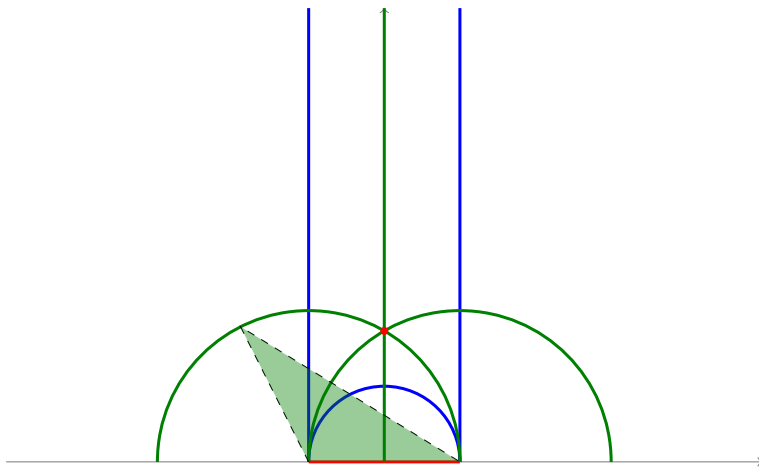
Isósceles



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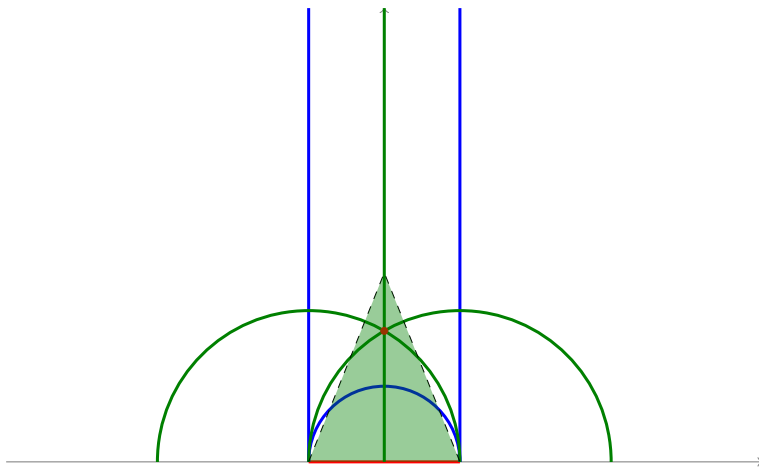
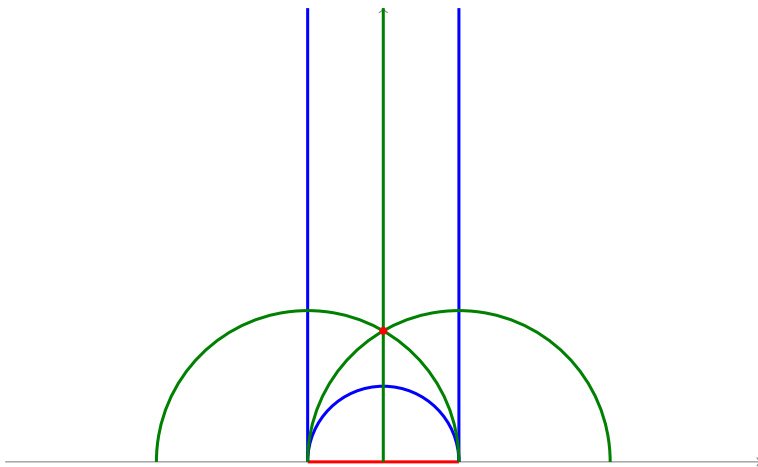
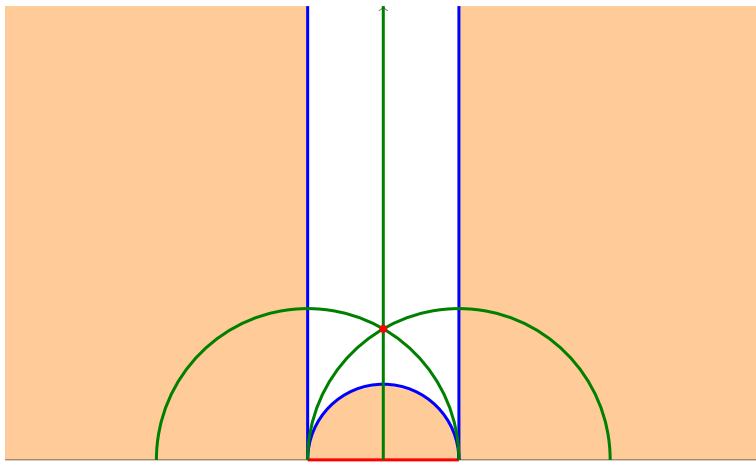


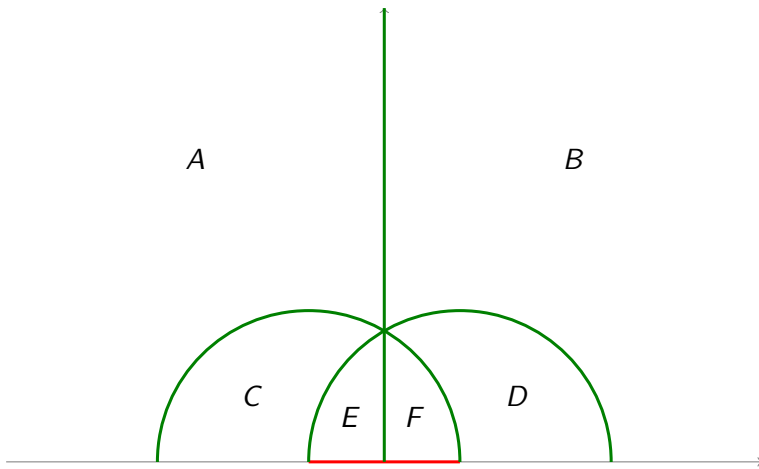
Figura completa



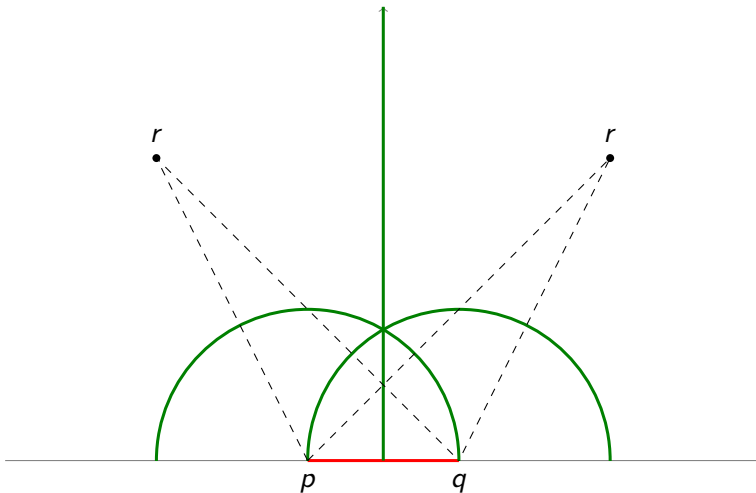
Curiosidade: acutângulos e obtusângulos



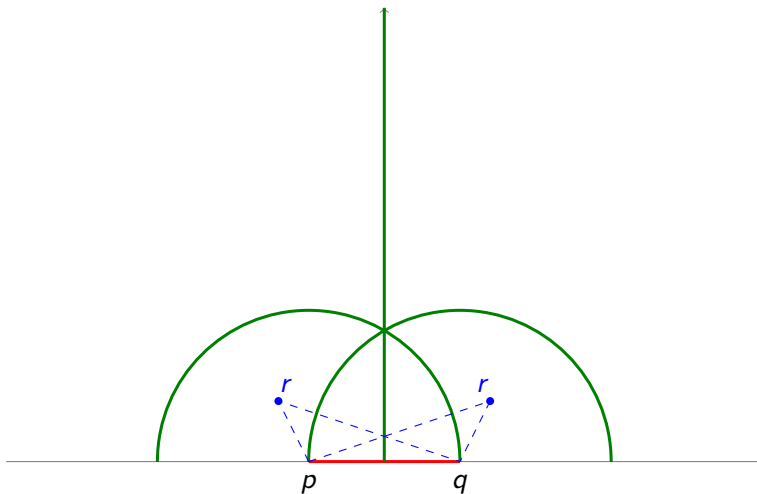
Redundâncias



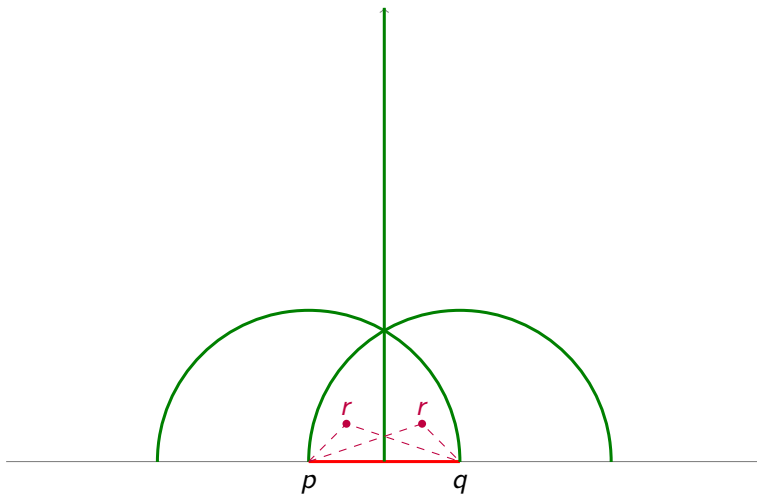
Redundâncias



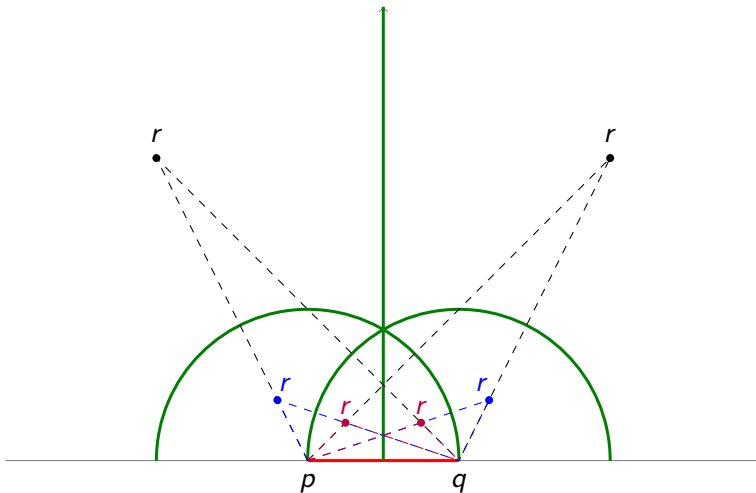
Redundâncias



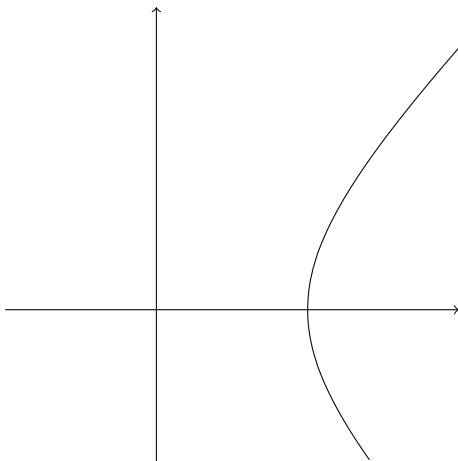
Redundâncias



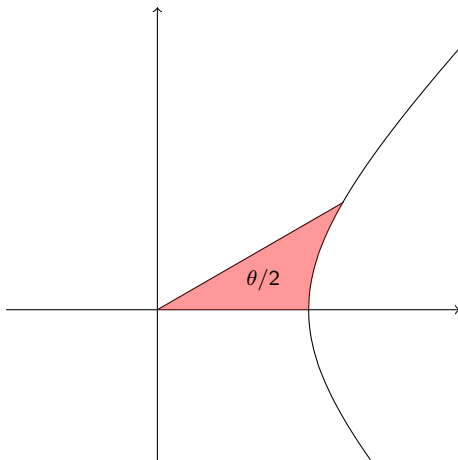
Redundâncias



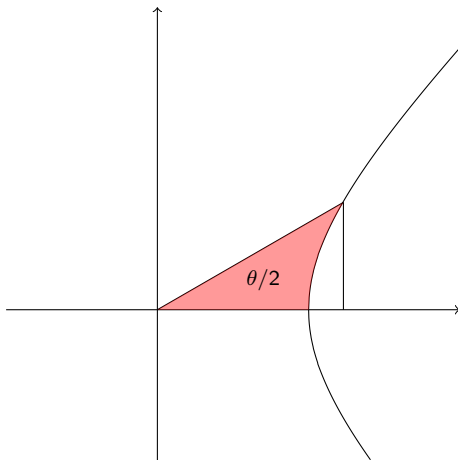
Cosseno hiperbólico



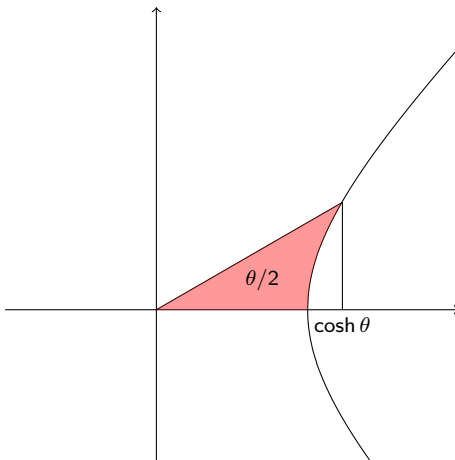
Cosseno hiperbólico



Cosseno hiperbólico



Cosseno hiperbólico



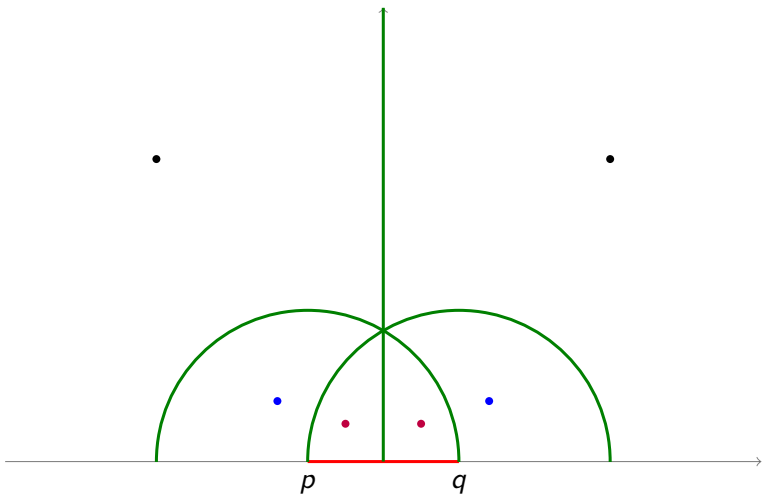
Distâncias

Se $x = (x_1, x_2)$ e $y = (y_1, y_2)$, então

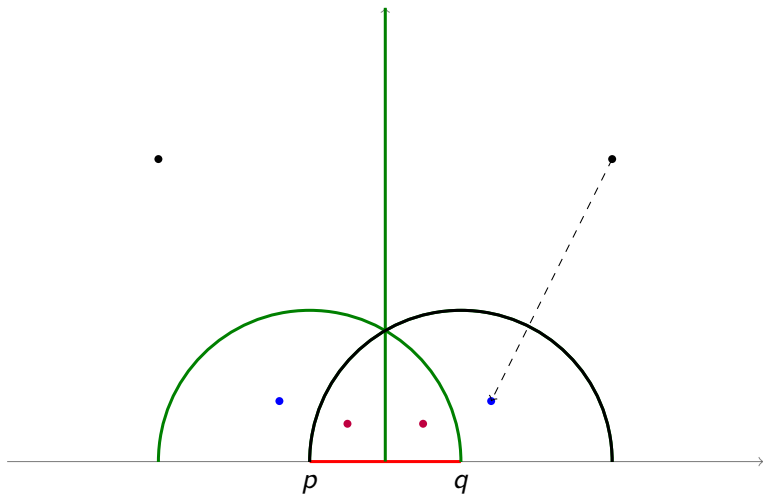
$$d_E(x, y) = \sqrt{|x_1 - y_1|^2 + |x_2 - y_2|^2}$$

$$d_H(x, y) = \cosh^{-1} \left(1 + \frac{d_E(x, y)^2}{2x_2y_2} \right)$$

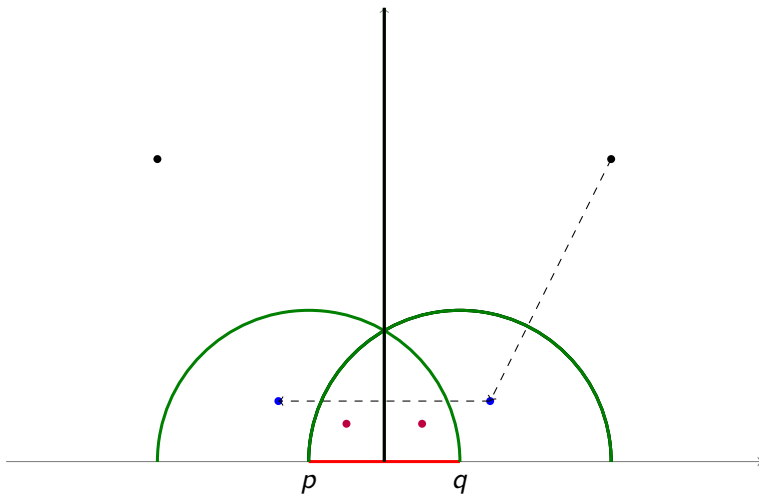
Redundâncias



Redundâncias



Redundâncias



- ① Curvatura
- ② Espaço das retas orientadas
- ③ Espaço dos triângulos
- ④ Referências

Referências

